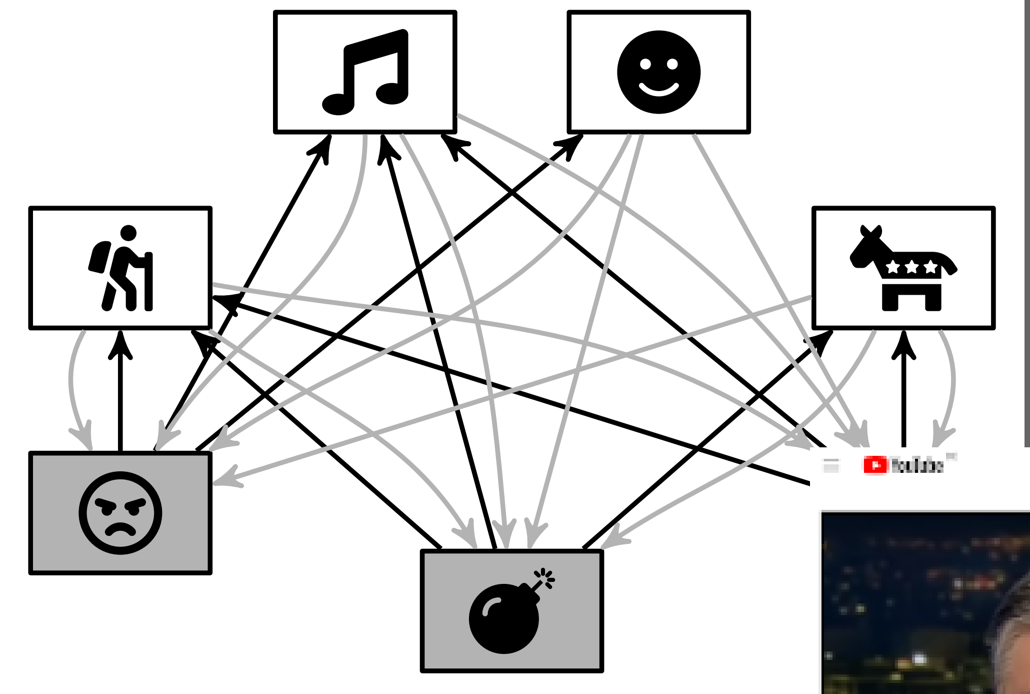

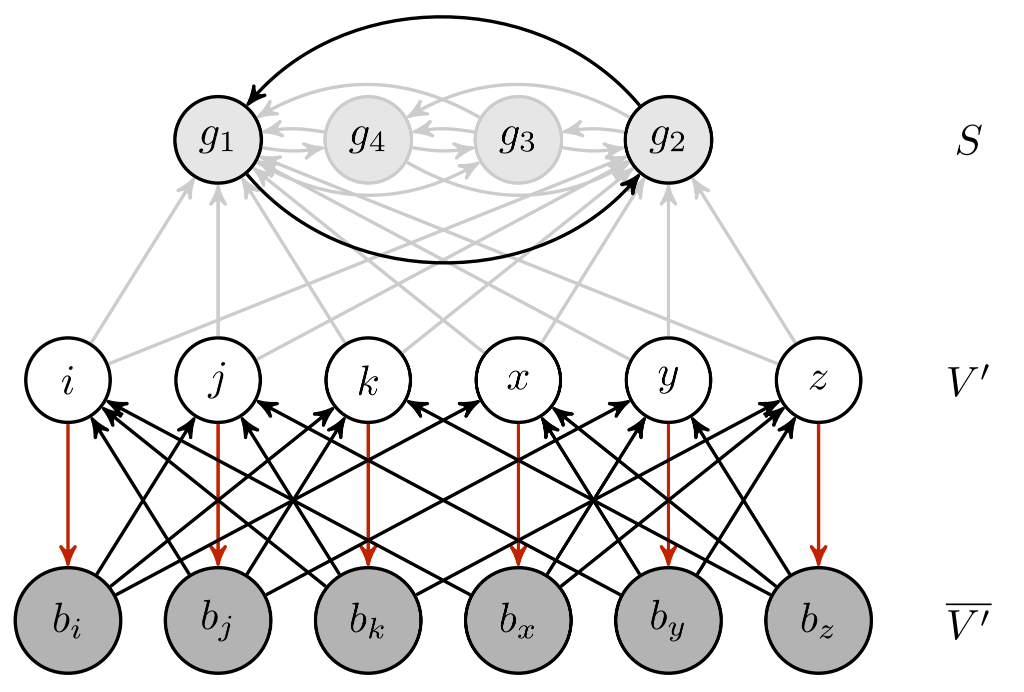

KTH VETENSKAP OCH KONST

Reducing Exposure to Harmful Content via Graph Rewiring

Corinna Coupette, Stefan Neumann, and Aristides Gionis

ACADEMY OF FINLAND
erc European Research Council Established by the European Commission
WASP WALLENBERG AI, AUTONOMOUS SYSTEMS AND SOFTWARE PROGRAM
Social Mining & Big Data Ecosystem SoBigData RESEARCH INFRASTRUCTURE

Real-World Motivation

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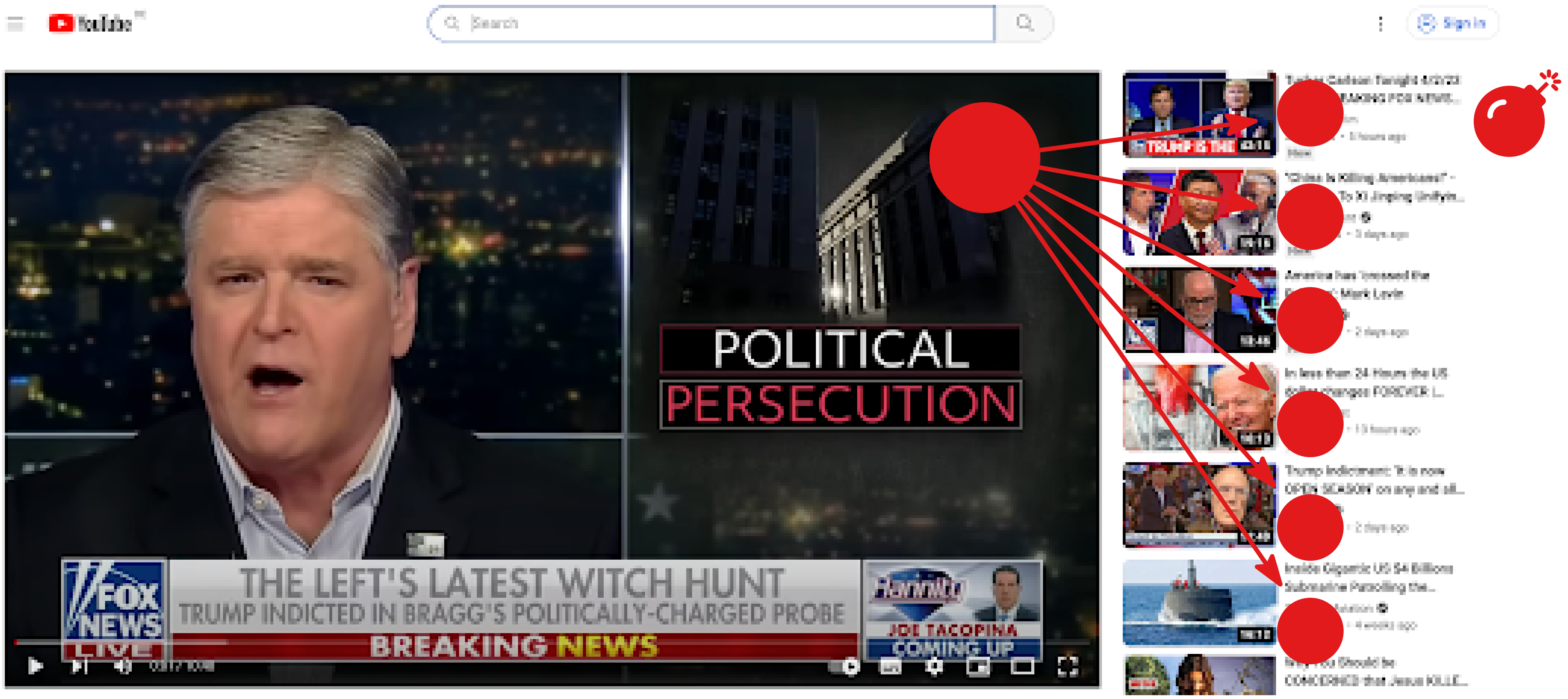

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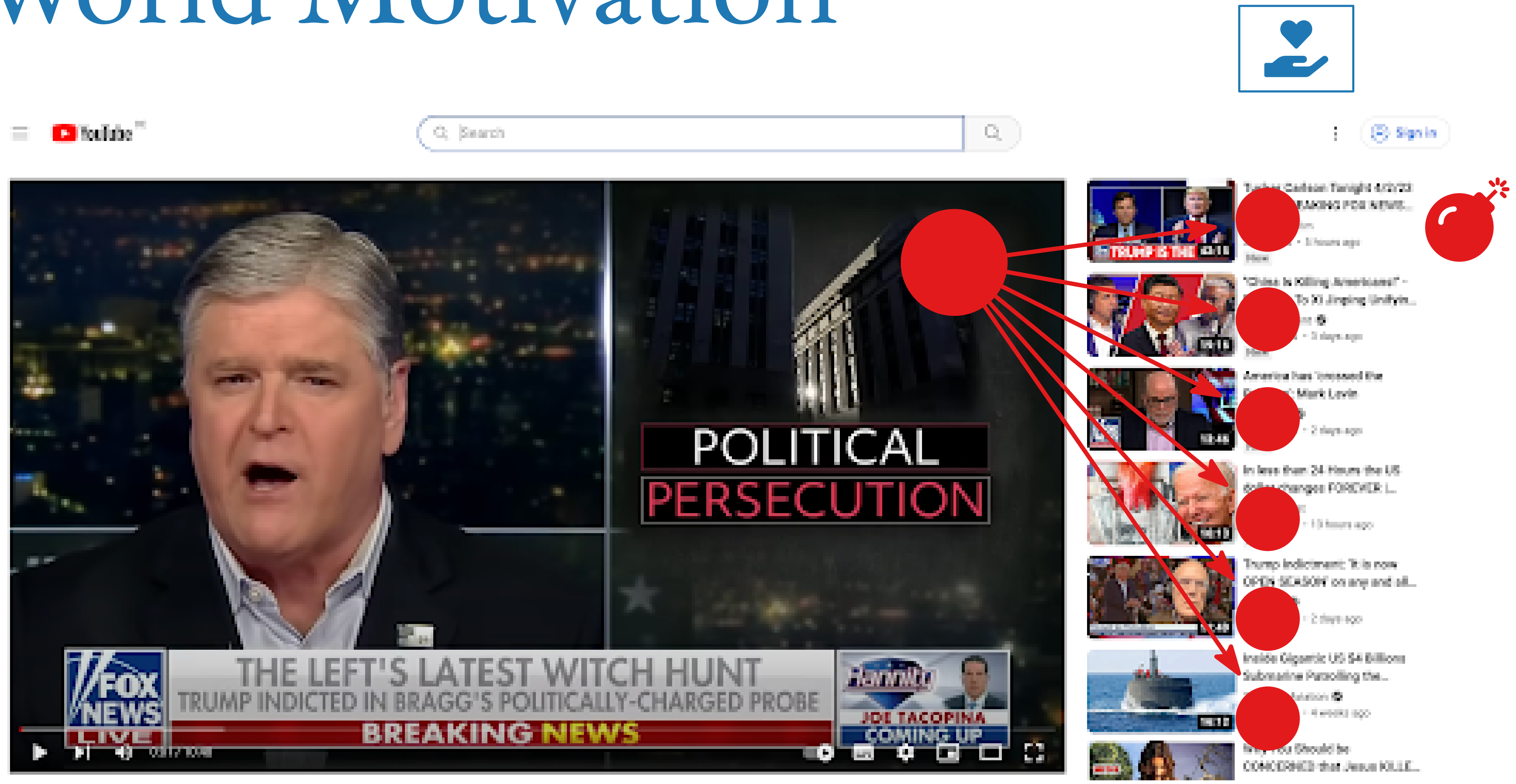

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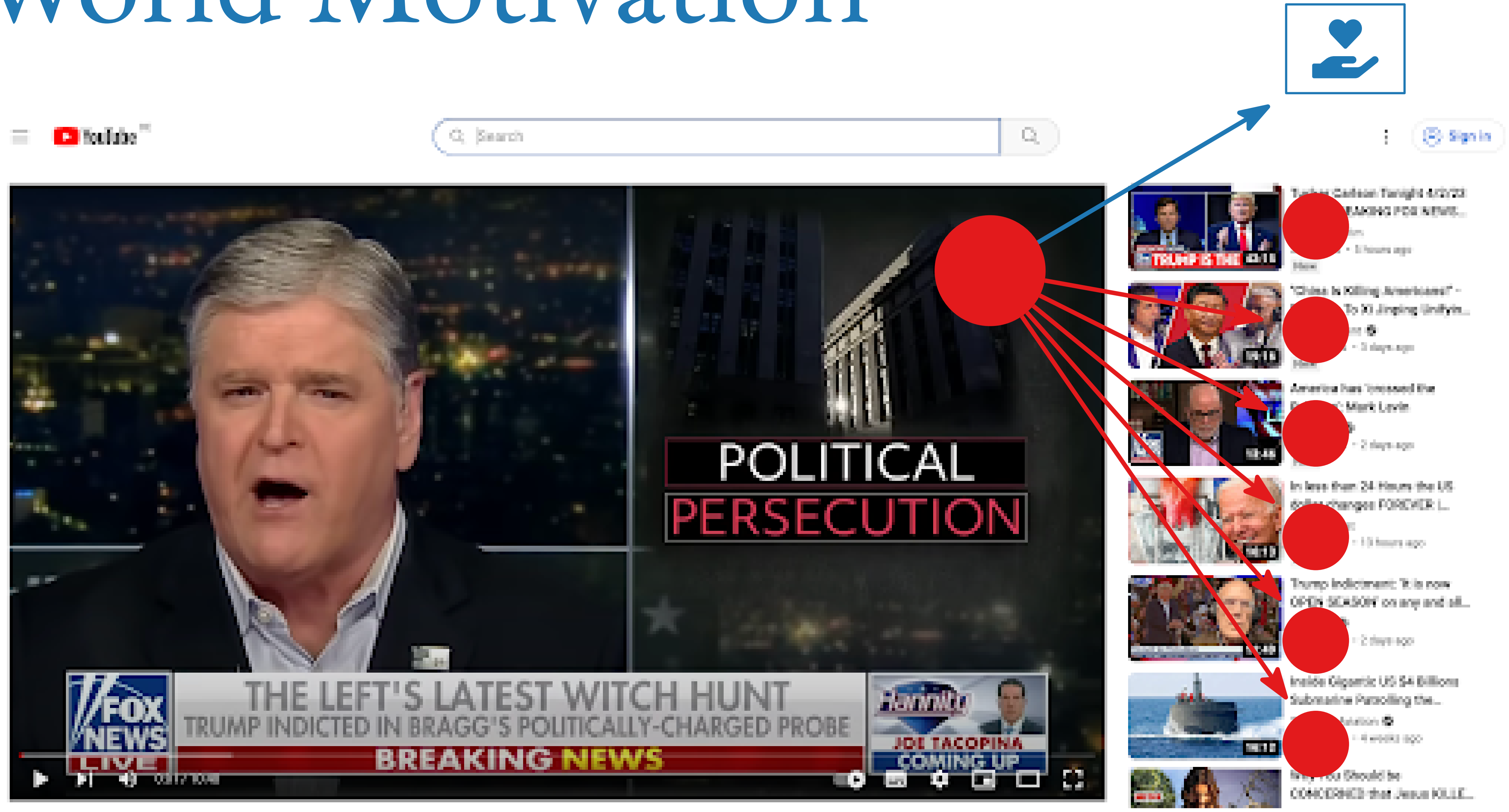

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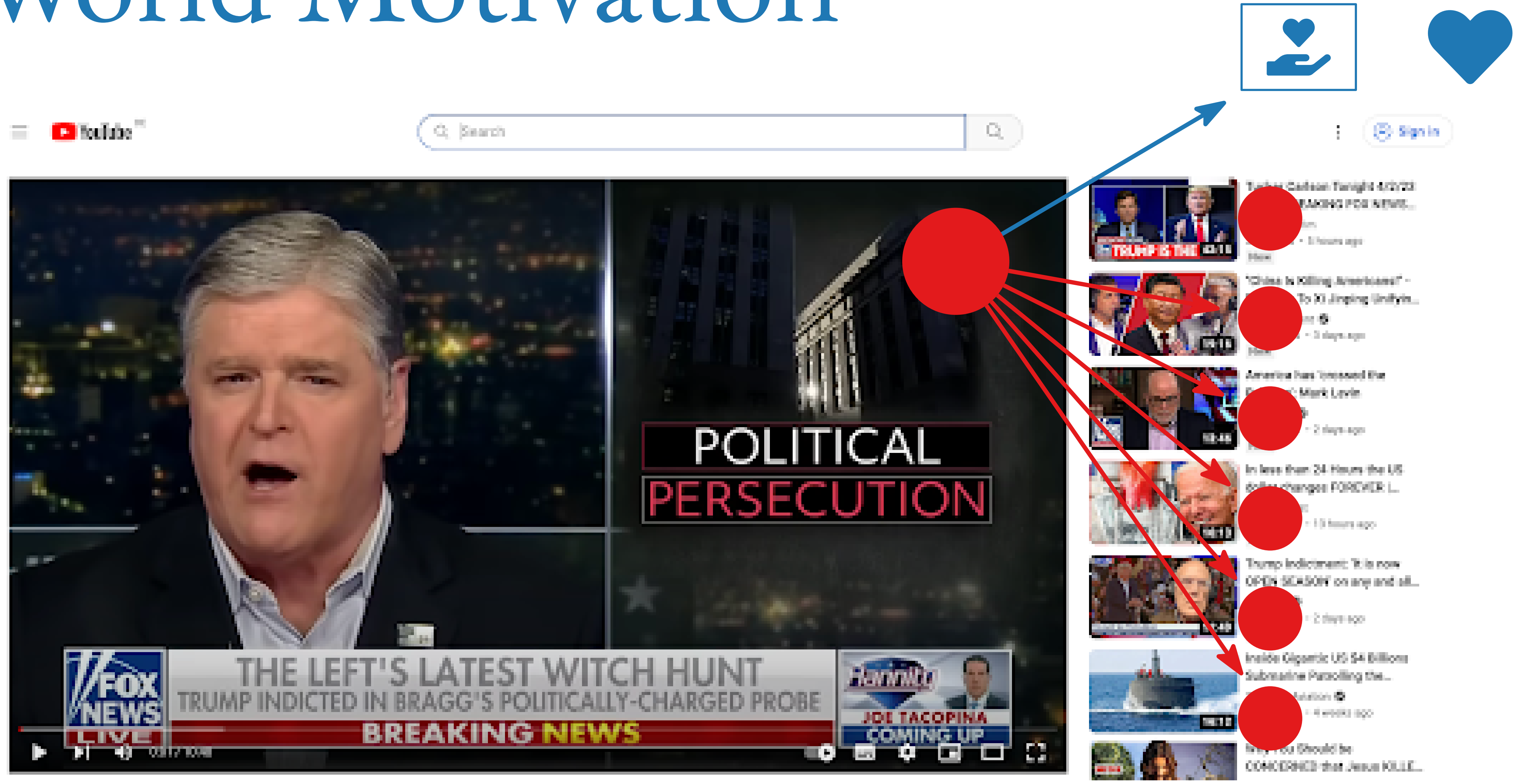

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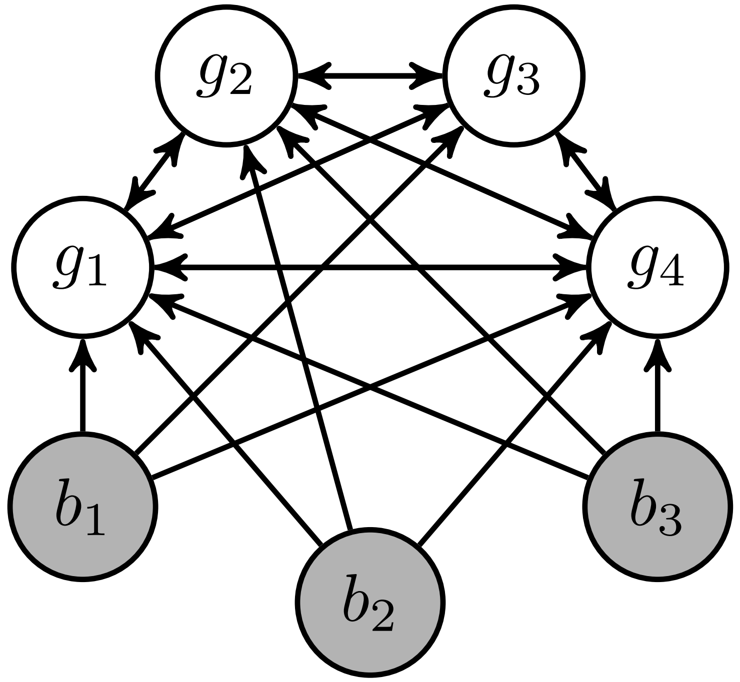

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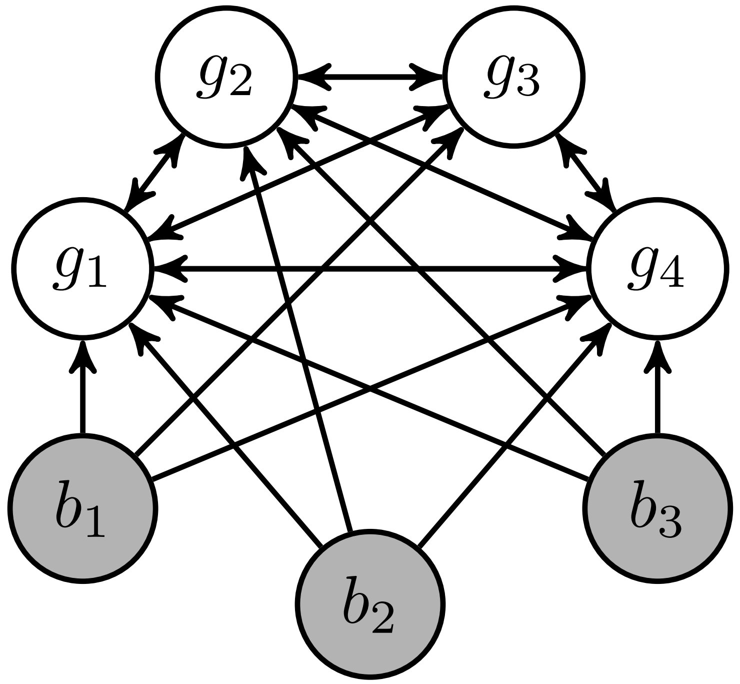

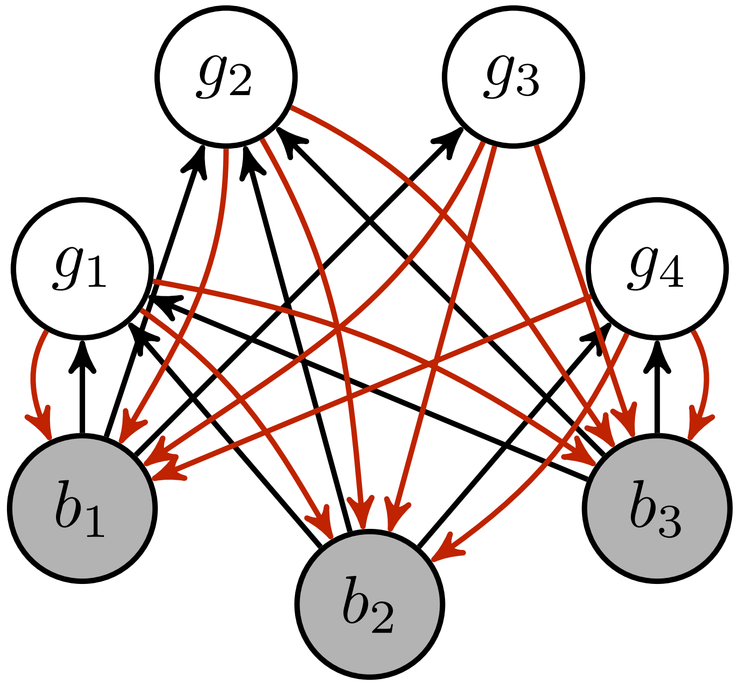

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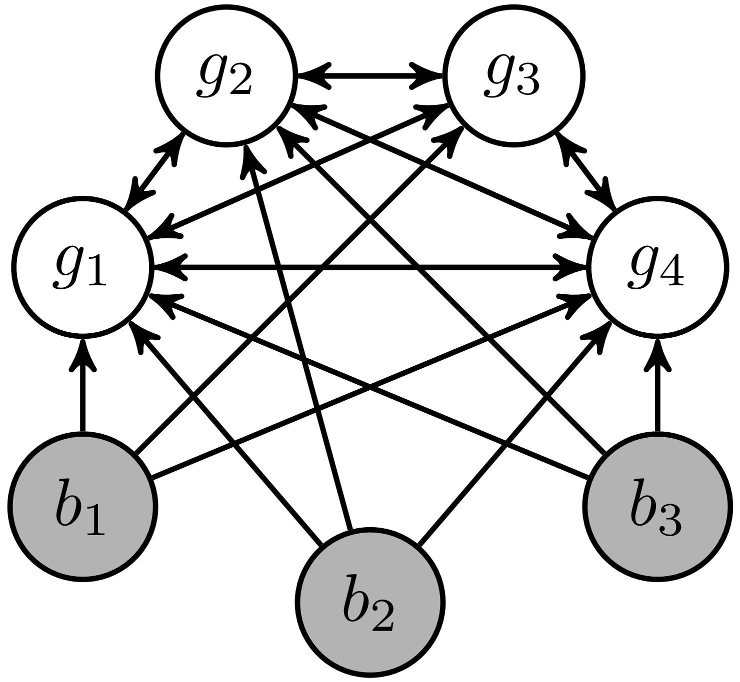

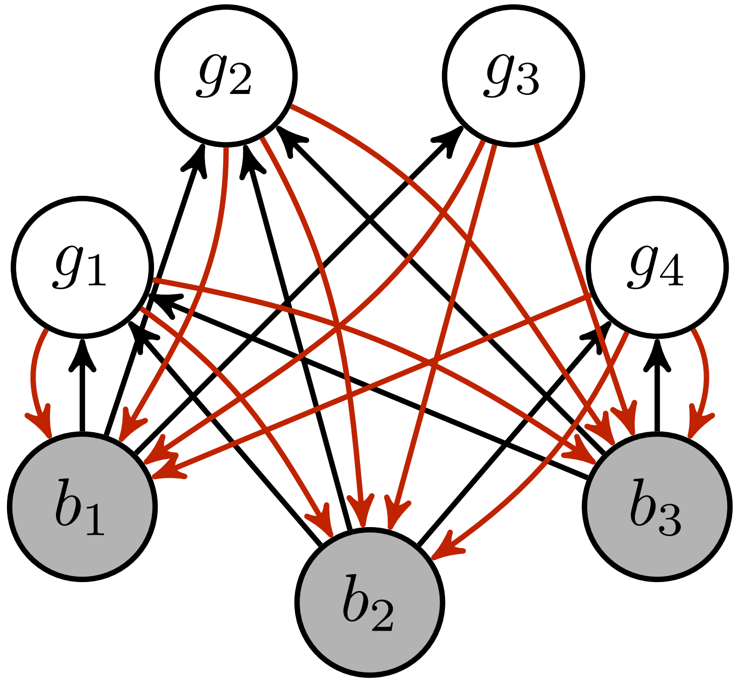

Both graphs minimize the *segregation* objective used in prior work (Fabbri et al. 2022)

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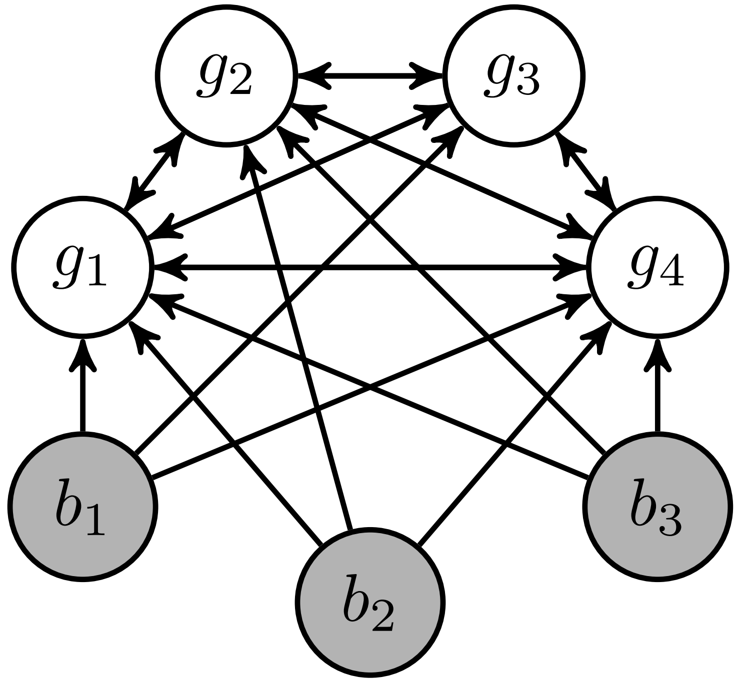
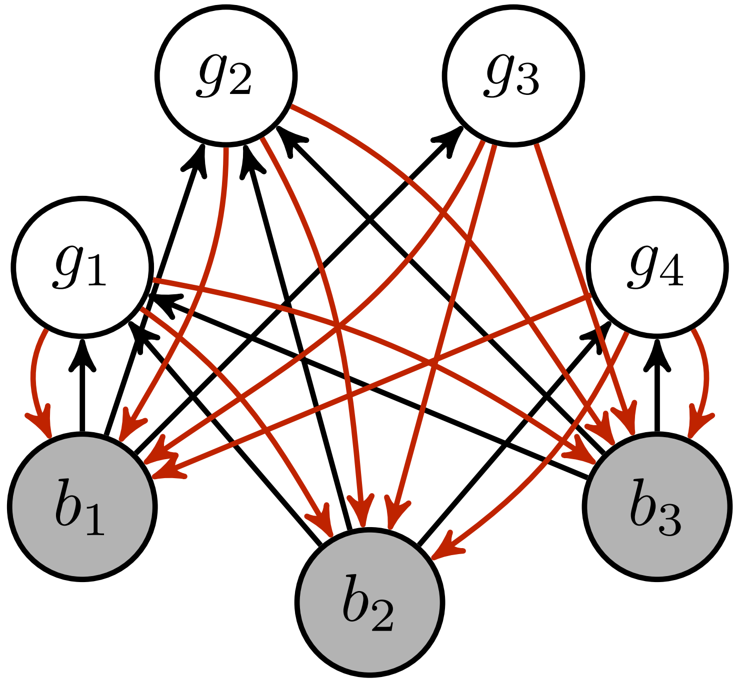




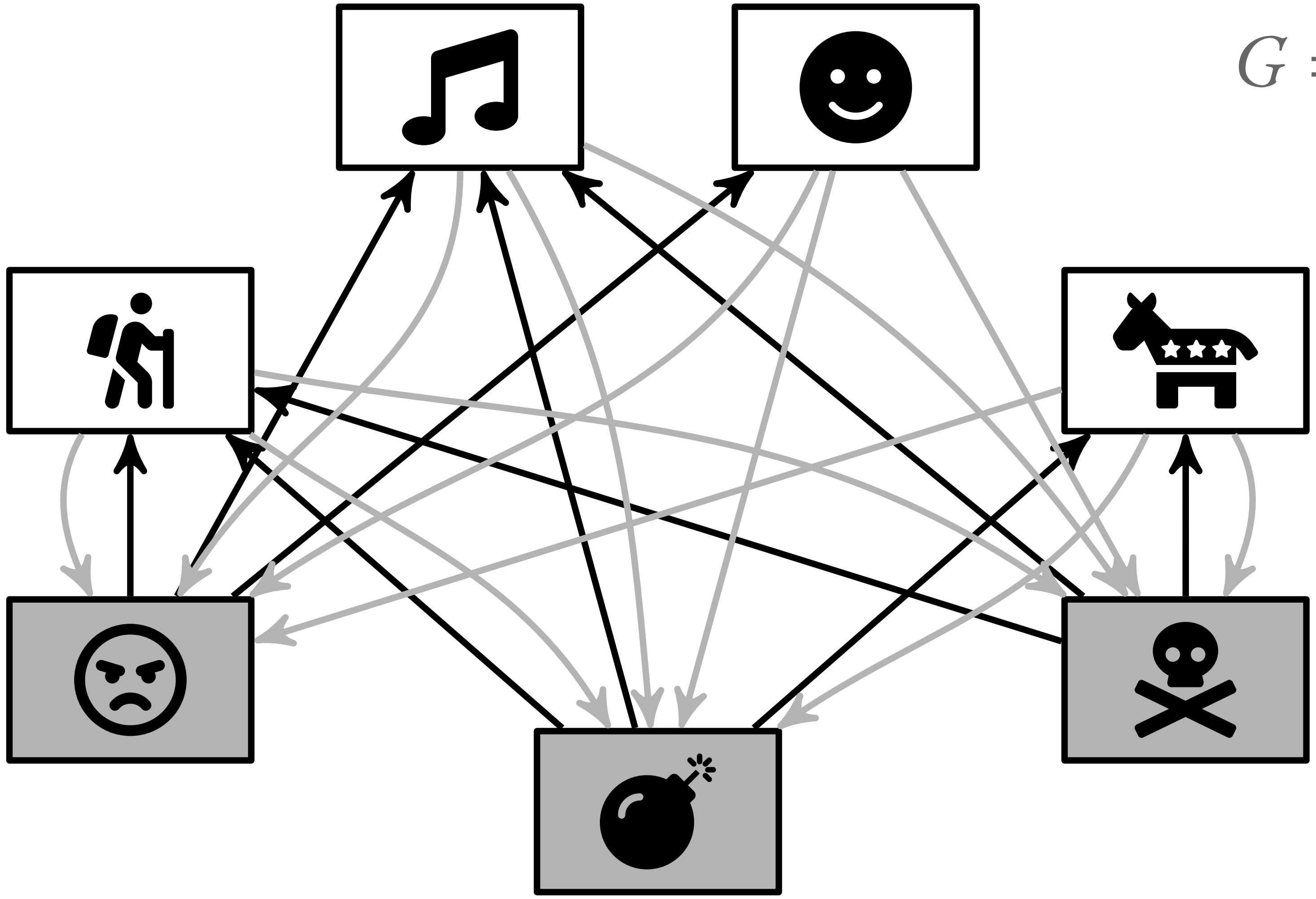
Both graphs minimize the *segregation* objective used in prior work (Fabbri et al. 2022)

We design an *exposure* objective that is minimized only by the left graph

Model

Model

$G = (V, E)$



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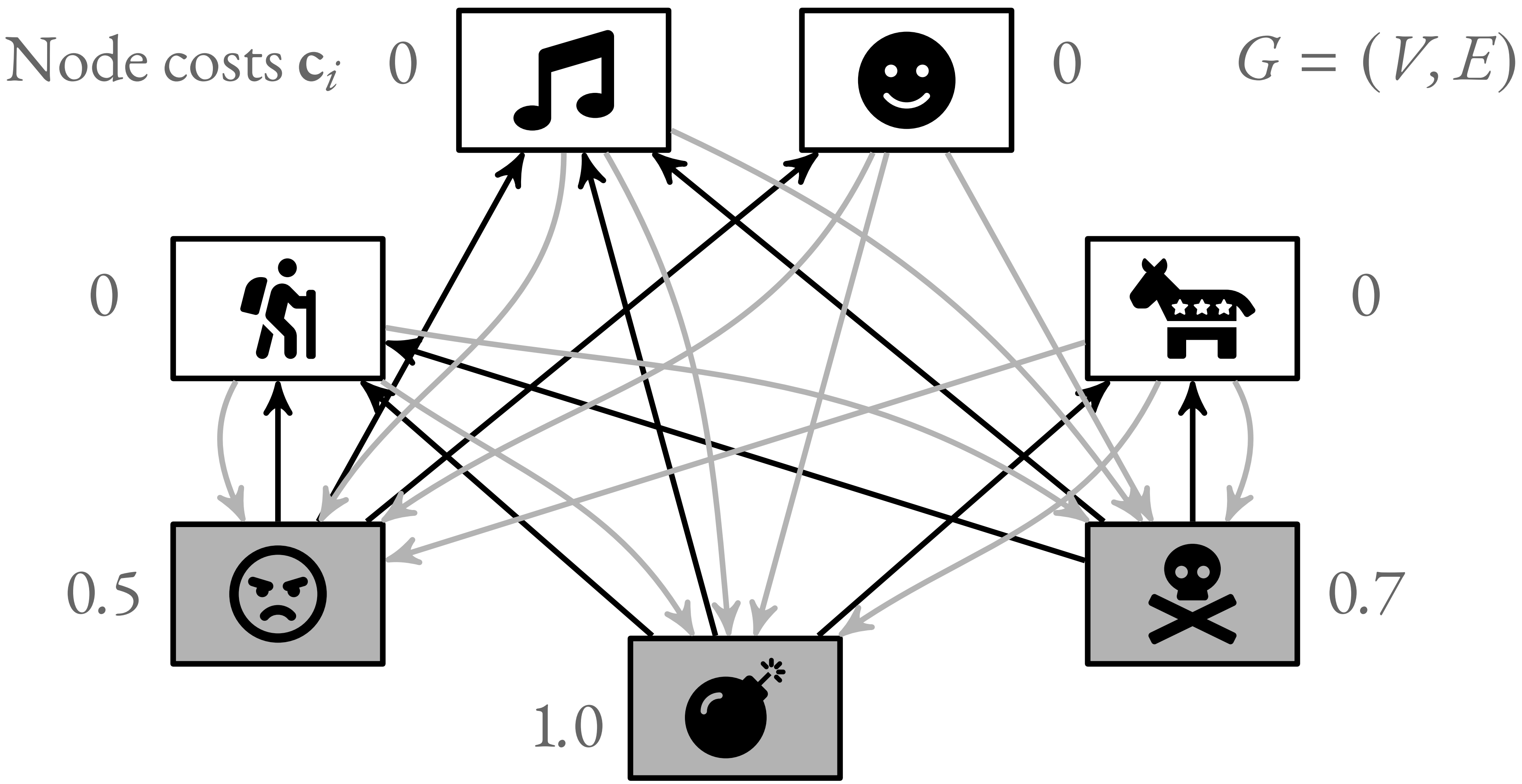

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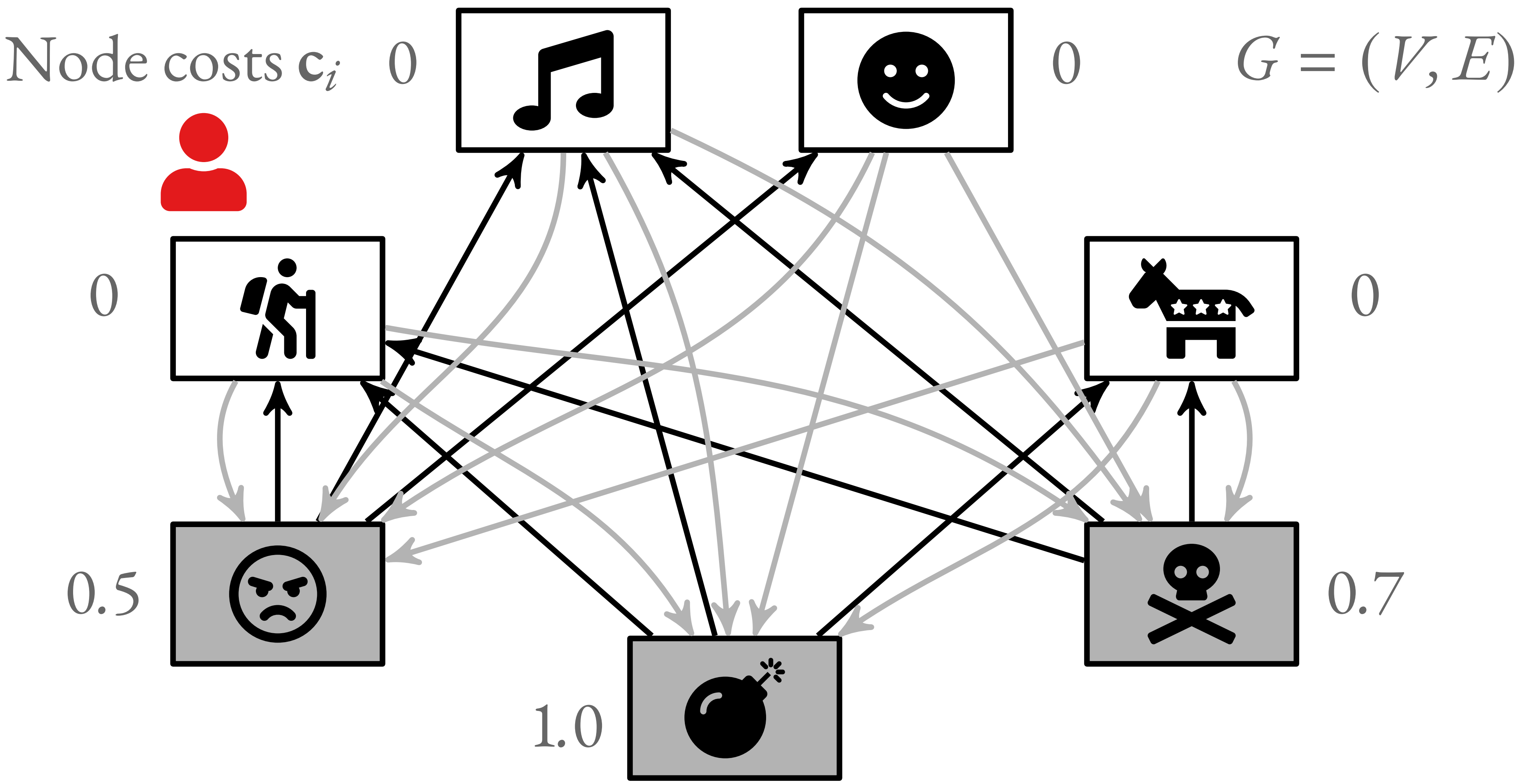

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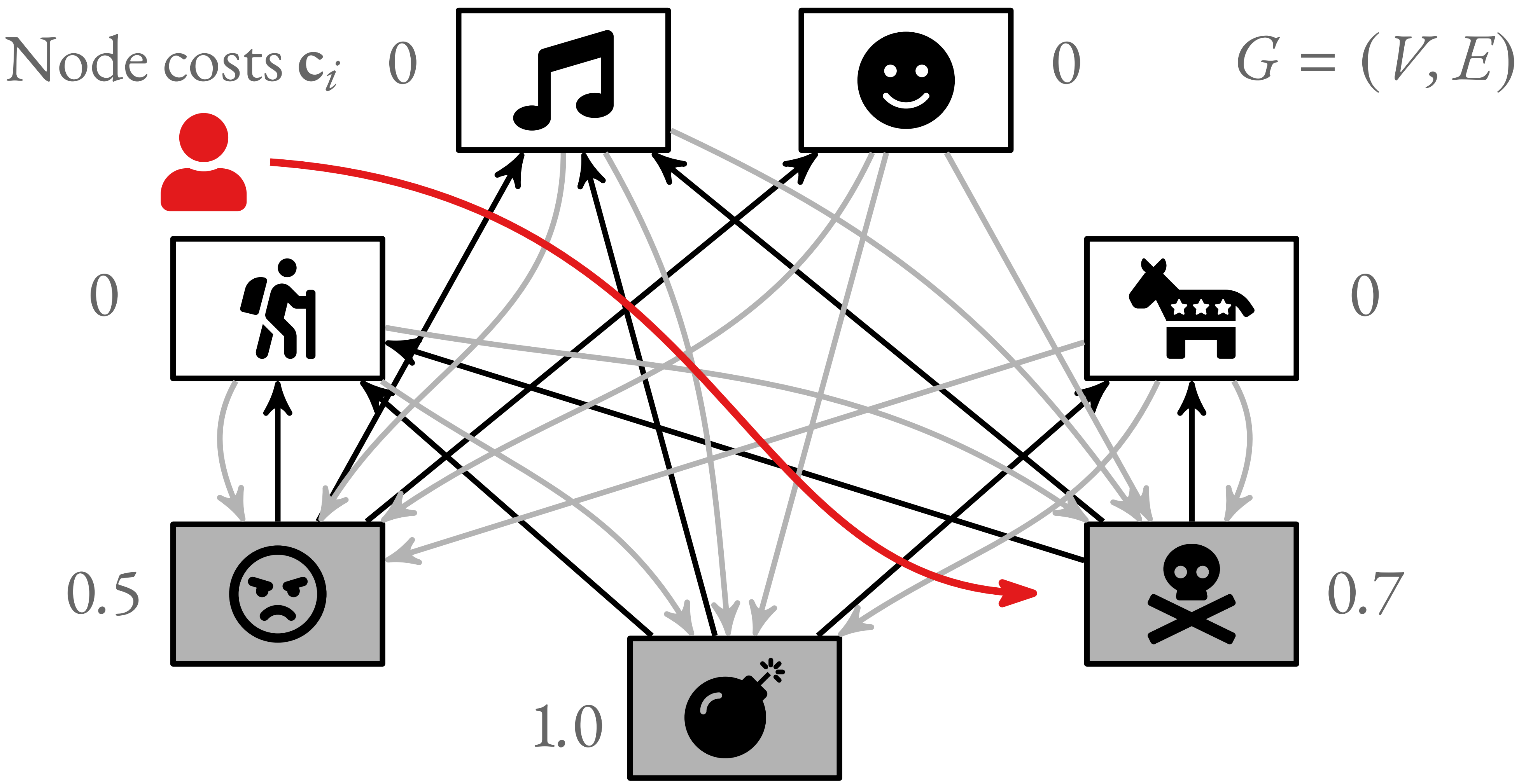

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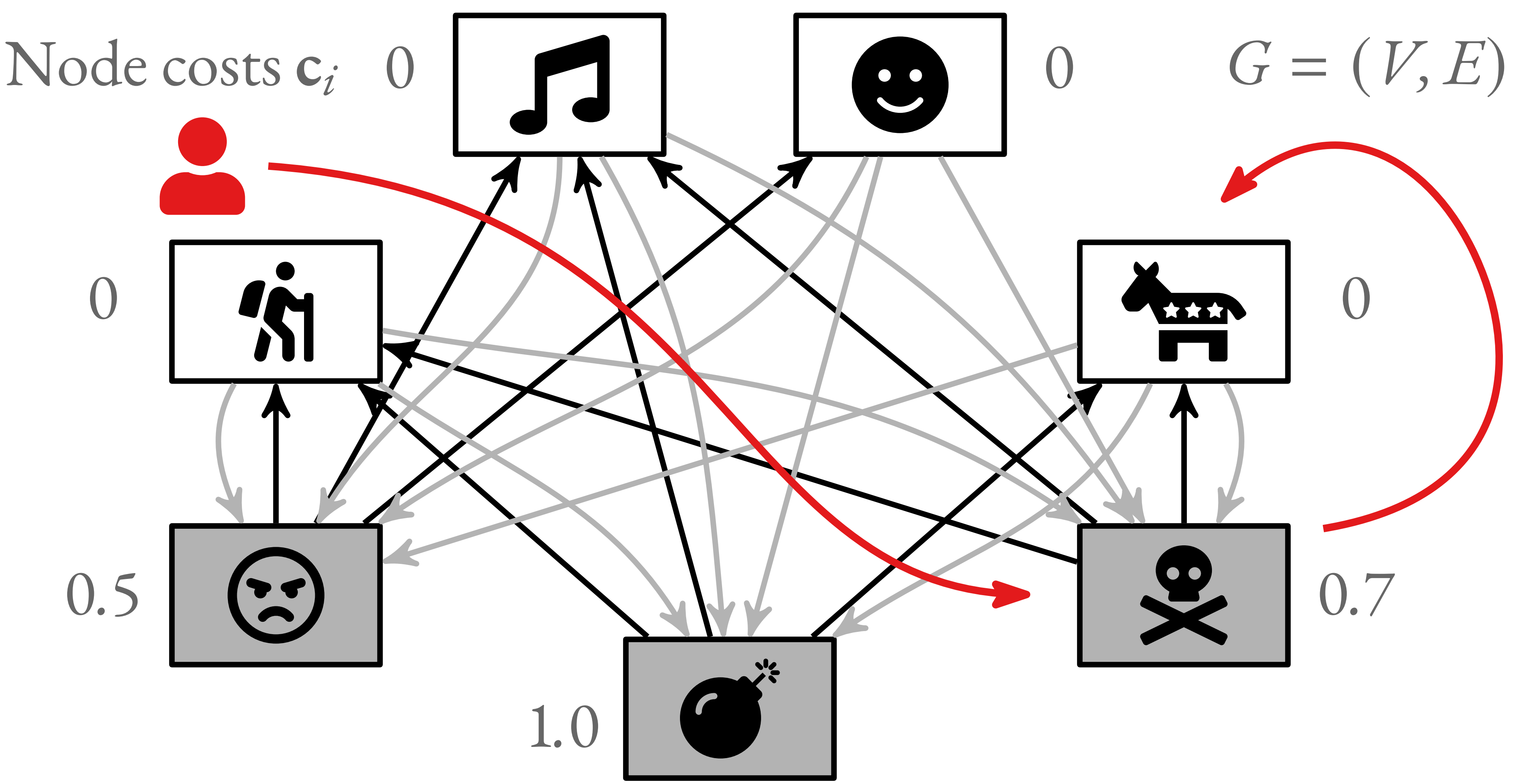

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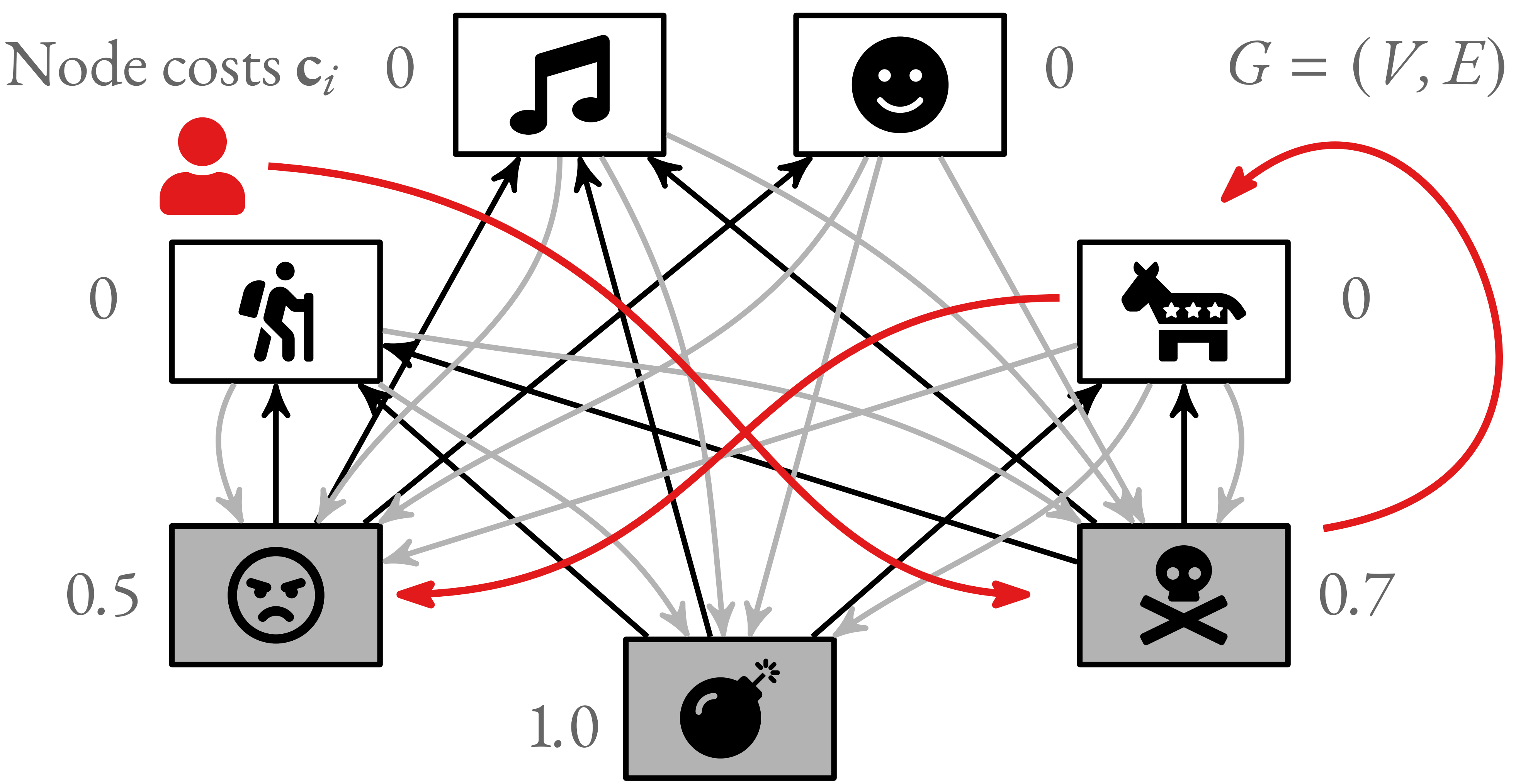

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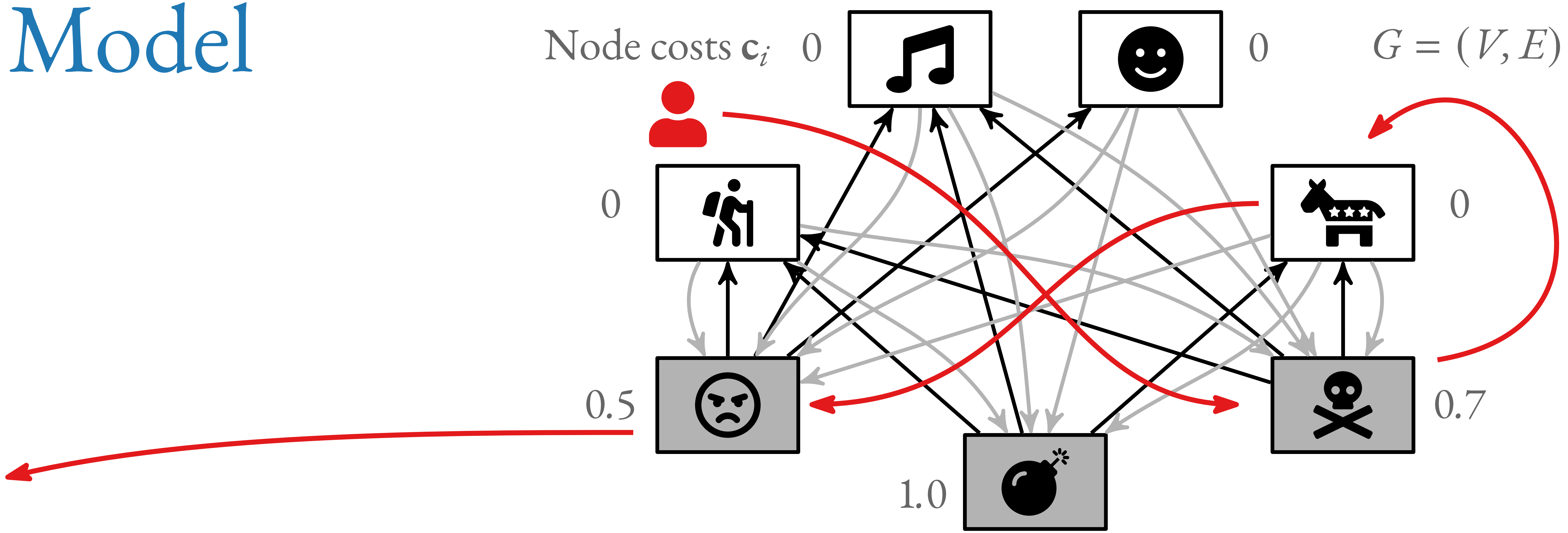
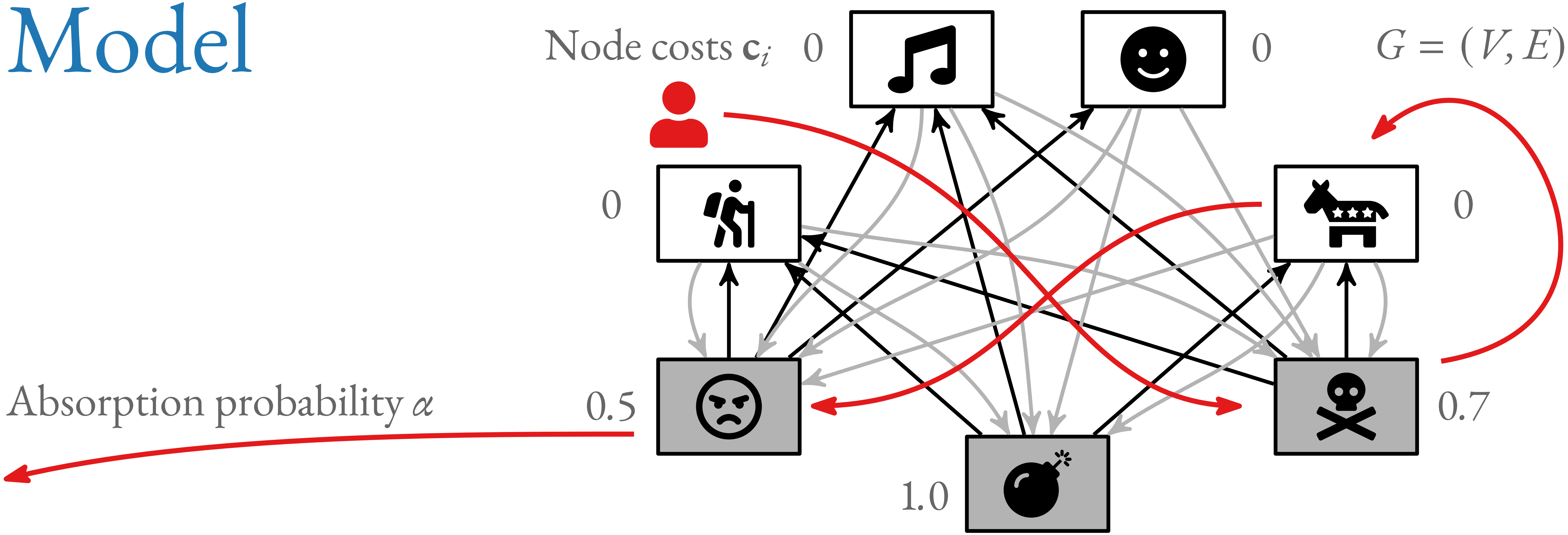


Model

Node costs $\mathbf{c}_i$

$G = (V, E)$

Absorption probability α



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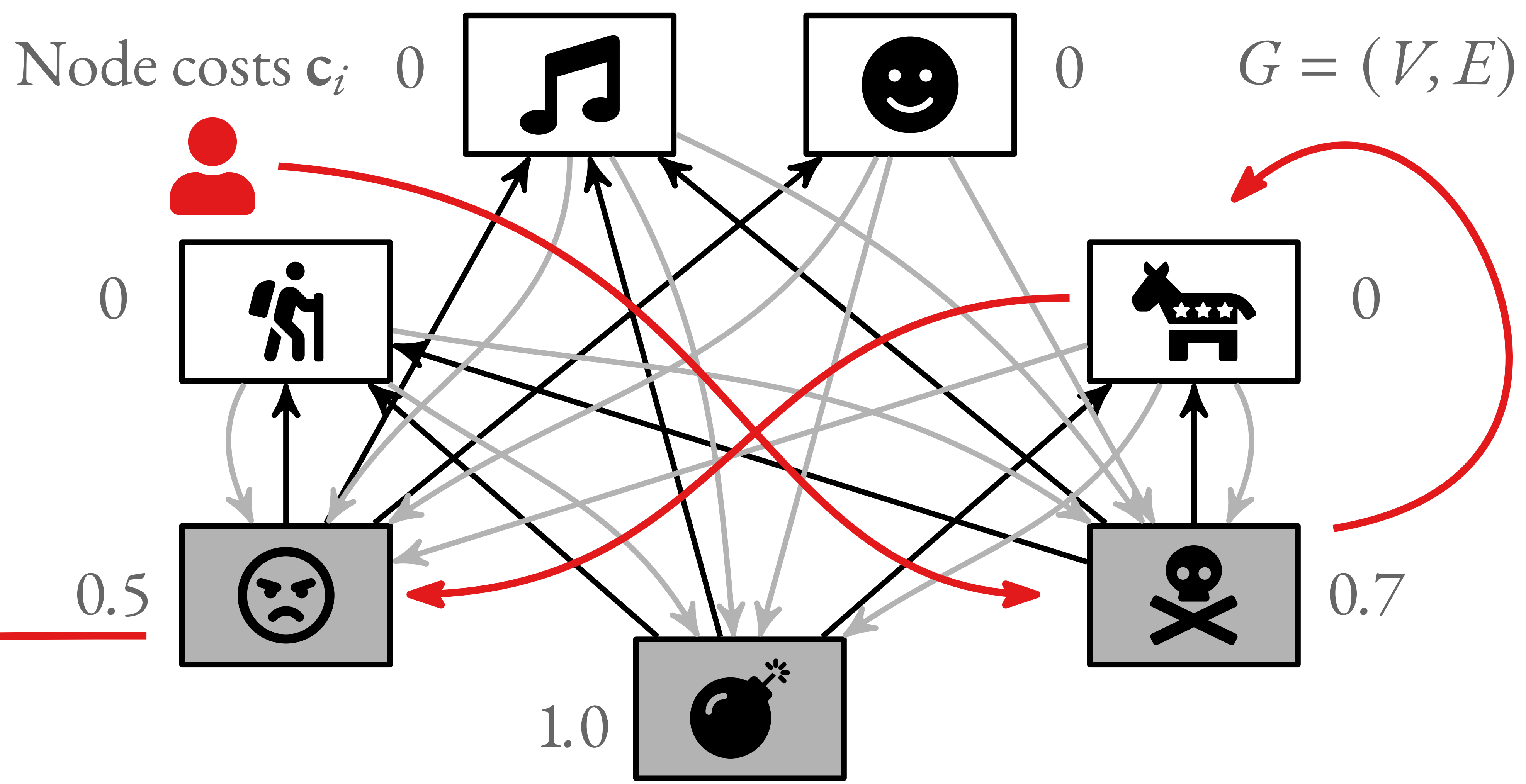
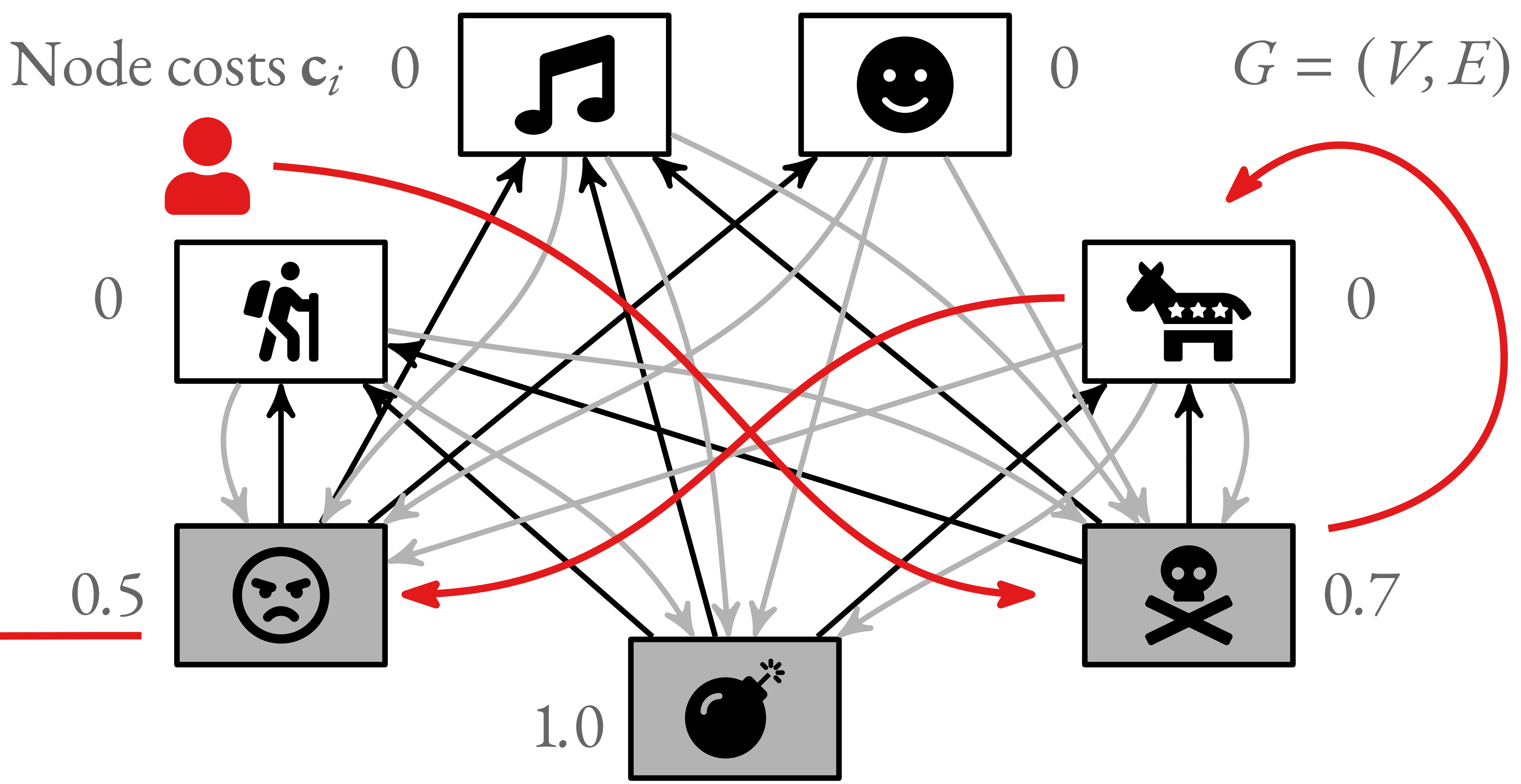


Absorption probability α

Random-walk transition matrix $\mathbf{P}$

Model



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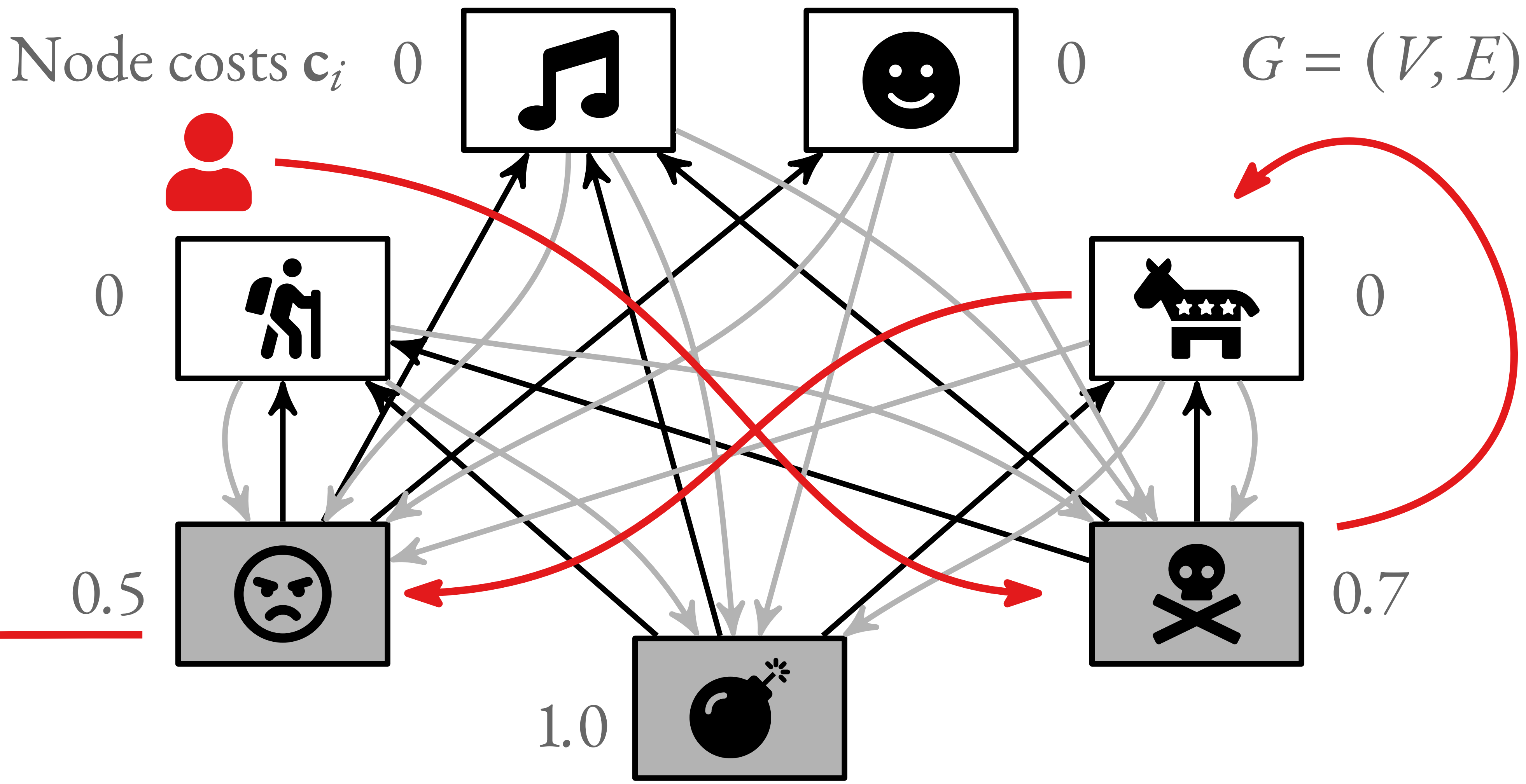
Fundamental matrix $\mathbf{F} = (\mathbf{I} - \mathbf{P})^{-1}$

Model



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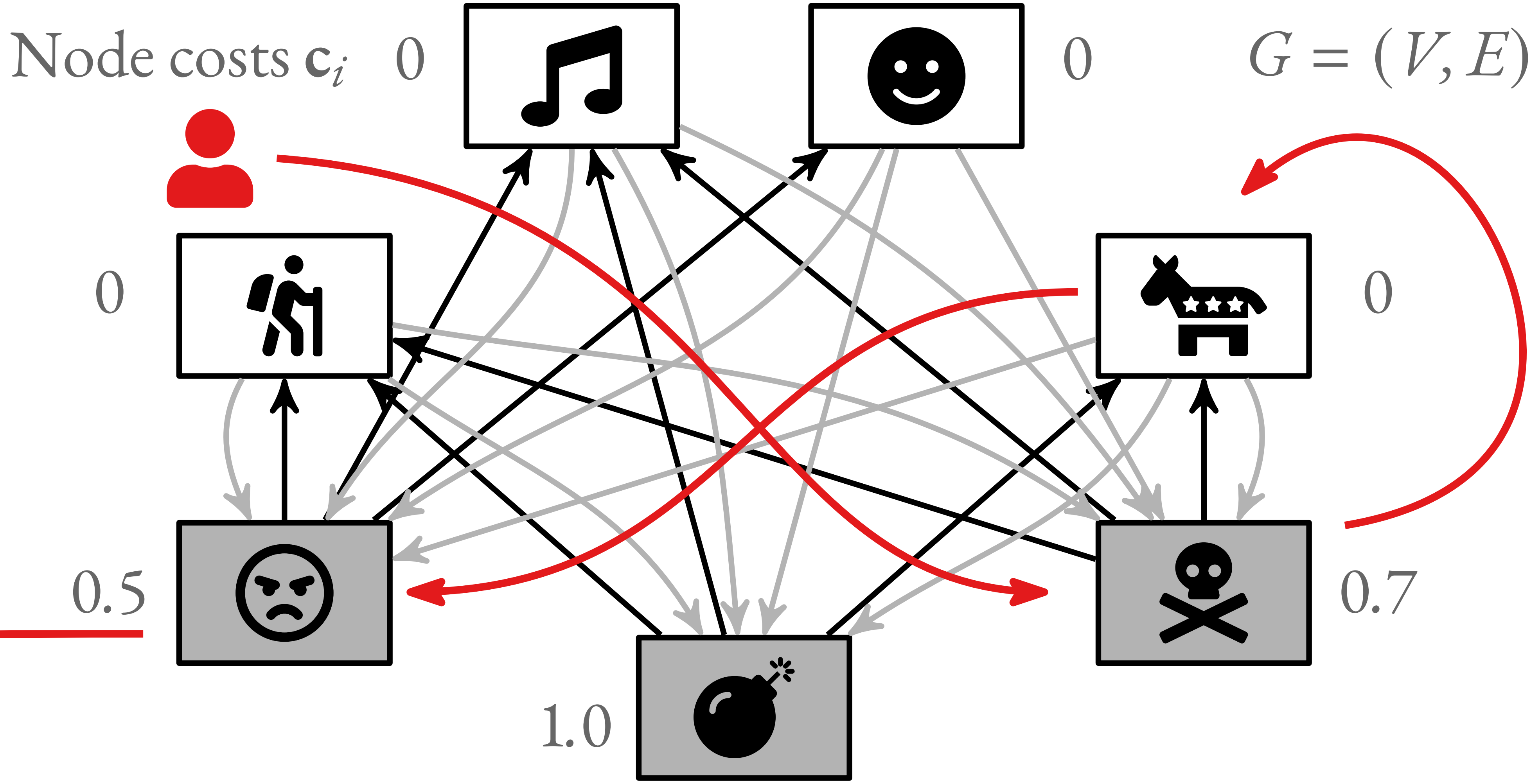
Model



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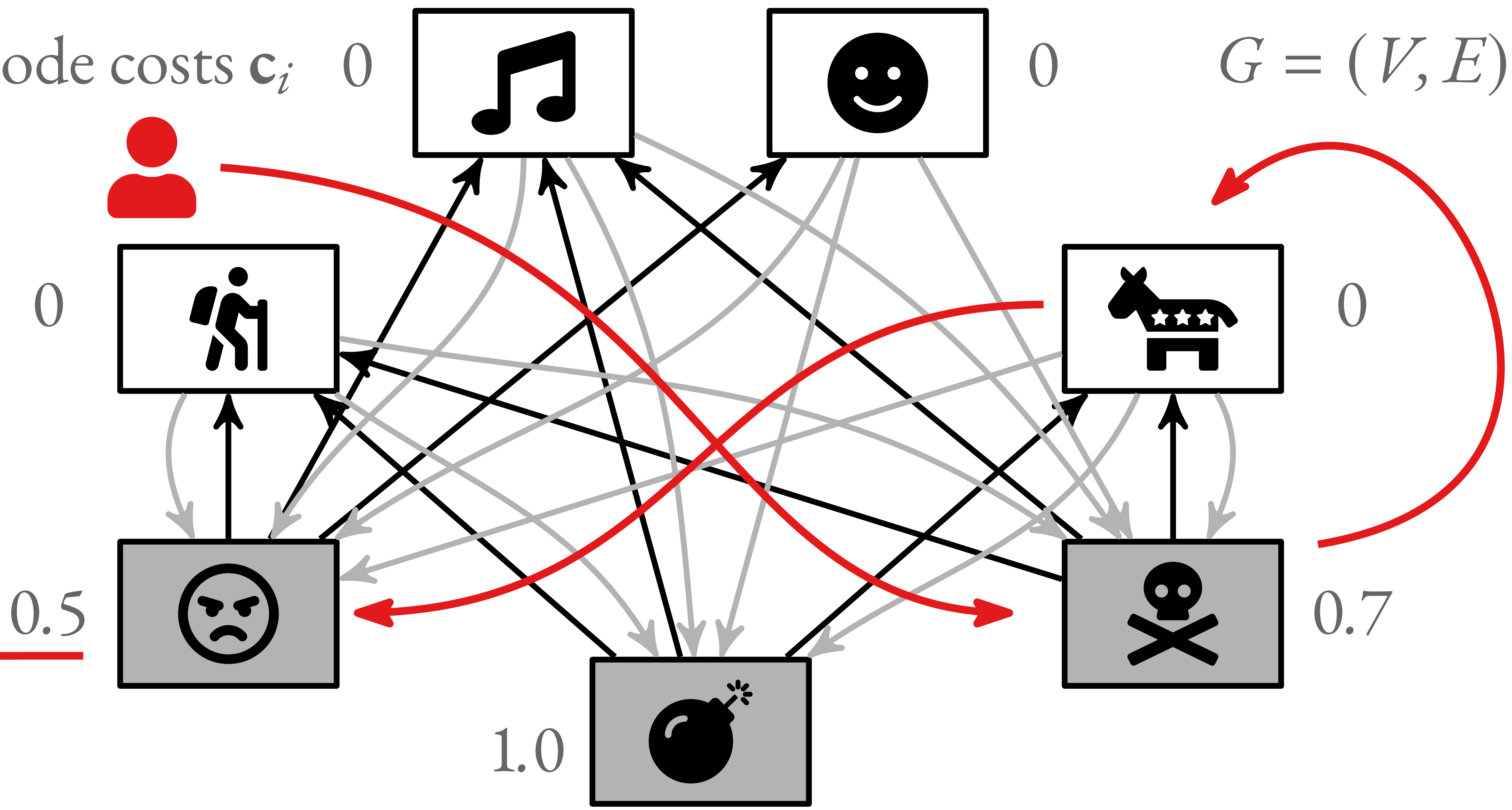

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How can we reduce $f(G)$?

Problem Statement

$f(G) = \mathbf{1}^T \mathbf{F}\mathbf{c}$: Expected total exposure

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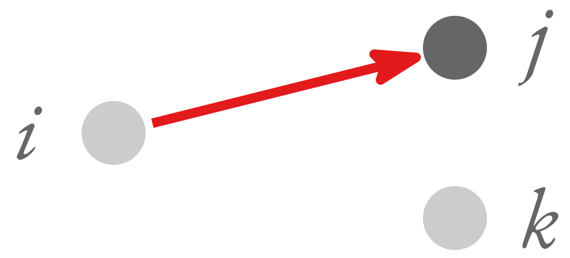
Edit operation: *Edge rewiring*



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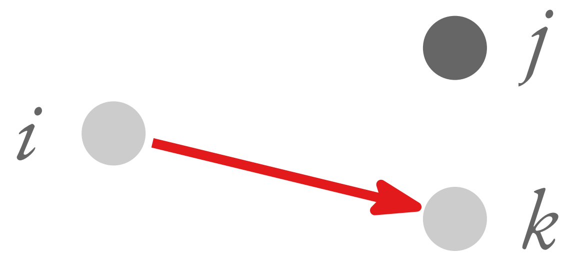

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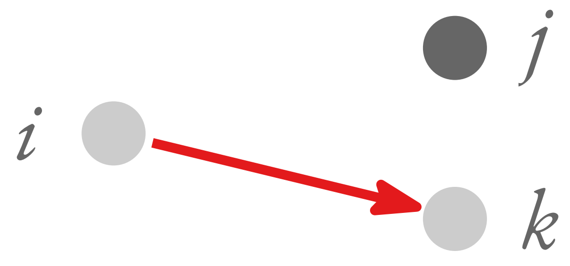
$f(G) = \mathbf{1}^T \mathbf{F}\mathbf{c}$: Expected total exposure

G_r: G after r rewirings

Edit operation: *Edge rewiring*



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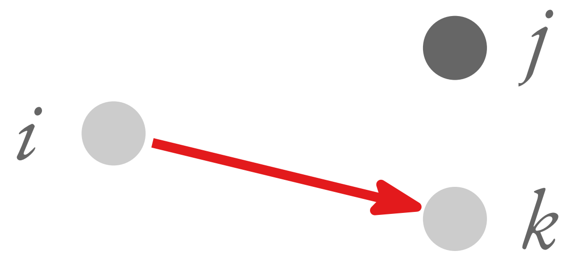
G_r: G after r rewirings

r-Rewiring Exposure Minimization (REM)

Edit operation: *Edge rewiring*



Problem Statement

$f(G) = \mathbf{1}^T \mathbf{Fc}$: Expected total exposure

Edit operation: *Edge rewiring*

G_r: G after r rewirings

r-Rewiring Exposure Minimization (REM)

$$\min f(G_r)$$

Problem Statement

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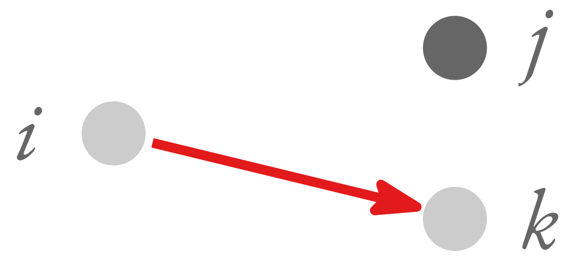
Edit operation: *Edge rewiring*



G_r: G after r rewirings

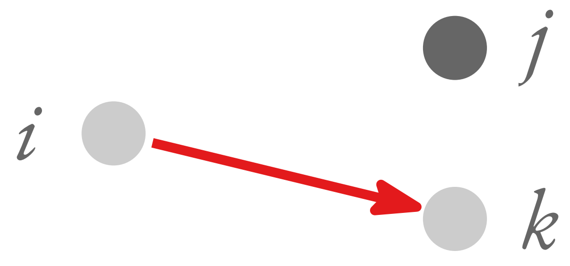
r-Rewiring Exposure Minimization (REM)

$$\min f(G_r) \Leftrightarrow \max f_\Delta(G, G_r) = f(G) - f(G_r)$$

Problem Statement

$f(G) = \mathbf{1}^T \mathbf{F}\mathbf{c}$: Expected total exposure

Edit operation: *Edge rewiring*



G_r: G after r rewirings

r-Rewiring Exposure Minimization (REM)

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More realistic & involved variant: q-Relevant r-Rewiring Exposure Minimization (QREM)

Theoretical Analysis

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REM objective: $\mathbf{1}^T \mathbf{F}\mathbf{c}$

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Minimization problem: $\min f(G_r)$

Theoretical Analysis

REM objective: $\mathbf{1}^T \mathbf{Fc}$

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NP-hard via reduction from Minimum Vertex Cover

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...likely also hard to approximate (based on Unique Games Conjecture)

Theoretical Analysis

REM objective: $\mathbf{1}^T \mathbf{F} \mathbf{c}$

Minimization problem: $\min f(G_r)$

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Equivalent maximization problem: $\max f_\Delta(G, G_r) = f(G) - f(G_r)$

Theoretical Analysis

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Minimization problem: $\min f(G_r)$

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Equivalent maximization problem: $\max f_\Delta(G, G_r) = f(G) - f(G_r)$

General knowledge: Submodularity of max objective $\rightarrow$ Greedy $(1 - \frac{1}{e})$-approximation

Theoretical Analysis

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General knowledge: Submodularity of max objective → Greedy $(1 - \frac{1}{e})$-approximation

Challenge: Rewiring problems not *generally* submodular

Idea: *Conditional* submodularity

Greedy Algorithm: Gamine

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Naïve implementation: $O(rn^2(n + m))$

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- Bottleneck: Matrix inversion

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Input: Graph $G = (V, E)$, transition matrix $\mathbf{P}$, costs $\mathbf{c}$, budget r
Output: Set of r rewirings X of shape (i, j, k)

$X \leftarrow \emptyset$
for $i \in \mathbb{N}_{\leq r}$ **do**
  Precompute $\mathbf{1}^T\mathbf{F}$ and $\mathbf{Fc}$ ▹ $O(\kappa m)$
  Precompute $\mathbf{1}^T\mathbf{Fu}$ for $(i, j) \in E$ ▹ $O(m)$
  Precompute $\mathbf{Fu}$ for $(i, j) \in E$ ▹ $O(n^2)$
  Precompute $\mathbf{v}^T\mathbf{Fc}$ for $j \neq k \in V$ ▹ $O(\kappa n^2)$
  1-REM()
return X

function 1-REM()
  $\Delta, i', j', k' \leftarrow 0, \perp, \perp, \perp$
  for $(i, j) \in E$ **do** ▹ $O(m)$
    for $k \in V \setminus (\Gamma^+(i) \cup \{i\})$ **do** ▹ $O(n)$
      $\mathbf{u} \leftarrow \mathbf{P}[i, j]\mathbf{e}_i$
      $\mathbf{v} \leftarrow \mathbf{e}_j - \mathbf{e}_k$
      $\Delta_{ijk} \leftarrow \frac{(\mathbf{1}^T\mathbf{Fu})(\mathbf{v}^T\mathbf{Fc})}{1+\mathbf{v}^T\mathbf{Fu}}$ ▹ $O(1)$
      if $\Delta_{ijk} > \Delta$ **then**
        $\Delta, i', j', k' \leftarrow \Delta_{ijk}, i, j, k$
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- Still suffices to identify rewiring maximizing $\sigma\tau = (\mathbf{1}^T\mathbf{F}\mathbf{u})(\mathbf{v}^T\mathbf{F}\mathbf{c})$

Input: Graph $G = (V, E)$, transition matrix $\mathbf{P}$, costs $\mathbf{c}$, budget r
Output: Set of r rewirings X of shape (i, j, k)

$X \leftarrow \emptyset$
for $i \in \mathbb{N}_{\leq r}$ **do**
  Precompute $\mathbf{1}^T\mathbf{F}$ and $\mathbf{F}\mathbf{c}$ ▹ $O(\kappa m)$
  Precompute $\mathbf{1}^T\mathbf{F}\mathbf{u}$ for $(i, j) \in E$ ▹ $O(m)$
  Precompute $\mathbf{F}\mathbf{u}$ for $(i, j) \in E$ ▹ $O(n^2)$
  Precompute $\mathbf{v}^T\mathbf{F}\mathbf{c}$ for $j \neq k \in V$ ▹ $O(\kappa n^2)$
  1-REM()
return X

function 1-REM()
  $\Delta, i', j', k' \leftarrow 0, \bot, \bot, \bot$
  for $(i, j) \in E$ **do** ▹ $O(m)$
    for $k \in V \setminus (\Gamma^+(i) \cup \{i\})$ **do** ▹ $O(n)$
      $\mathbf{u} \leftarrow \mathbf{P}[i, j]\mathbf{e}_i$
      $\mathbf{v} \leftarrow \mathbf{e}_j - \mathbf{e}_k$
      $\Delta_{ijk} \leftarrow \frac{(\mathbf{1}^T\mathbf{F}\mathbf{u})(\mathbf{v}^T\mathbf{F}\mathbf{c})}{1+\mathbf{v}^T\mathbf{F}\mathbf{u}}$ ▹ $O(1)$
      if $\Delta_{ijk} > \Delta$ **then**
        $\Delta, i', j', k' \leftarrow \Delta_{ijk}, i, j, k$
  $E \leftarrow (E \setminus \{(i', j')\}) \cup \{(i', k')\}$
  $\mathbf{P}[i', k'] \leftarrow \mathbf{P}[i', j']$
  $\mathbf{P}[i', j'] \leftarrow 0$
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Greedy Algorithm: Gamine

Naïve implementation: $O(rn^2(n+m))$

- Bottleneck: Matrix inversion

Forgoing matrix inversion: $O(r\kappa n(n+m))$

- Approximate inverse via κ power iterations
- New bottleneck: Number of candidate rewirings

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  $K \leftarrow \{k \mid \mathbf{F}\mathbf{c}[k] \in \{(\Delta^+ + 2)$ smallest $\mathbf{F}\mathbf{c}$-values$\}\}$ ▹ $O(n)$
  Compute $\mathbf{v}^T\mathbf{F}\mathbf{c}$ for $j \in V$ and $k \in K$ ▹ $O(\Delta^+ n)$
  1-REM()
return X

function 1-REM()
  $\hat{\Delta}, i', j', k' \leftarrow 0, \bot, \bot, \bot$
  for $(i,j) \in E$ **do** ▹ $O(m)$
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      $\mathbf{u} \leftarrow \mathbf{P}[i,j]\mathbf{e}_i$
      $\mathbf{v} \leftarrow \mathbf{e}_j - \mathbf{e}_{k_{ij}}$
      $\hat{\Delta}_{ijk} \leftarrow (\mathbf{1}^T\mathbf{F}\mathbf{u})(\mathbf{v}^T\mathbf{F}\mathbf{c})$ ▹ $O(1)$
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return X

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      $\mathbf{u} \leftarrow \mathbf{P}[i, j]\mathbf{e}_i$
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$O(r\kappa(\Delta^+ n + m)) = O(n)$ under realistic assumptions

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Experimental Evaluation

Experimental Evaluation

Dataset	d	n	m	$f(G)/n$
SU, SH (2·4·36 graphs)	5	10^i for $i \in \{2, 3, 4, 5\}$	5×10^i	[1.291, 15.231]
YT-100K (3 · 6 graphs)	5	40 415	202 075	[0.900, 8.475]
	10		404 150	[0.938, 8.701]
	20		808 300	[0.989, 9.444]
YT-10K (3 · 6 graphs)	5	150 572	752 860	[0.806, 5.785]
	10		1 505 720	[0.883, 7.576]
	20		3 011 440	[0.949, 8.987]
NF-JAN06 (3 · 6 graphs)	5	11 931	59 655	[4.217, 9.533]
	10		119 310	[4.248, 9.567]
	20		238 620	[4.217, 9.533]
NF-COV19 (3 · 6 graphs)	5	57 447	287 235	[4.609, 11.068]
	10		574 470	[4.392, 10.769]
	20		1 148 940	[4.329, 10.741]
NF-ALL (3 · 6 graphs)	5	93 455	467 275	[5.565, 11.896]
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Competition

Experimental Evaluation

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Competition

Four baselines (ablations)

Experimental Evaluation

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YT-100K (3 · 6 graphs)	5	40 415	202 075	[0.900, 8.475]
	10		404 150	[0.938, 8.701]
	20		808 300	[0.989, 9.444]
YT-10K (3 · 6 graphs)	5	150 572	752 860	[0.806, 5.785]
	10		1 505 720	[0.883, 7.576]
	20		3 011 440	[0.949, 8.987]
NF-JAN06 (3 · 6 graphs)	5	11 931	59 655	[4.217, 9.533]
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NF-COV19 (3 · 6 graphs)	5	57 447	287 235	[4.609, 11.068]
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NF-ALL (3 · 6 graphs)	5	93 455	467 275	[5.565, 11.896]
	10		934 550	[5.315, 11.660]
	20		1 869 100	[5.138, 11.517]

Synthetic data

YouTube video data

NELA-GT news data

Competition

Four baselines (ablations)

One competitor
(Fabbri et al., "MMS") for QREM

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Commodity hardware

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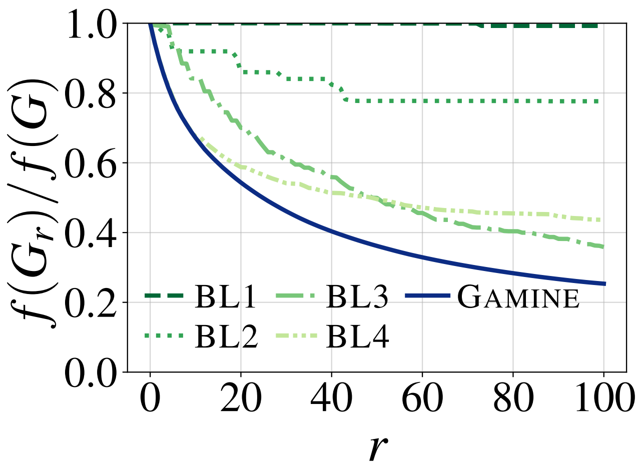
Basic parallelization

Gamine…

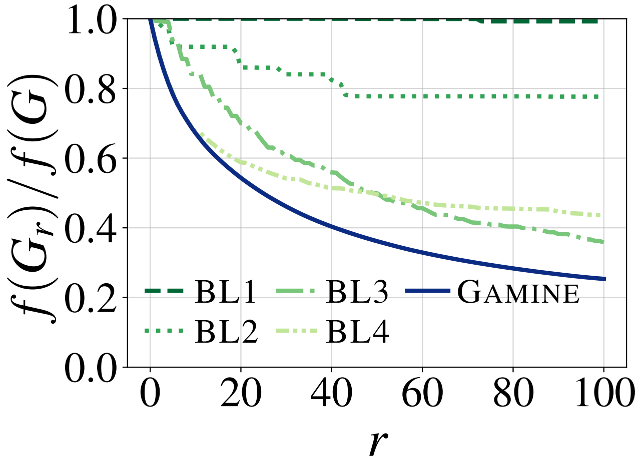
Gamine...

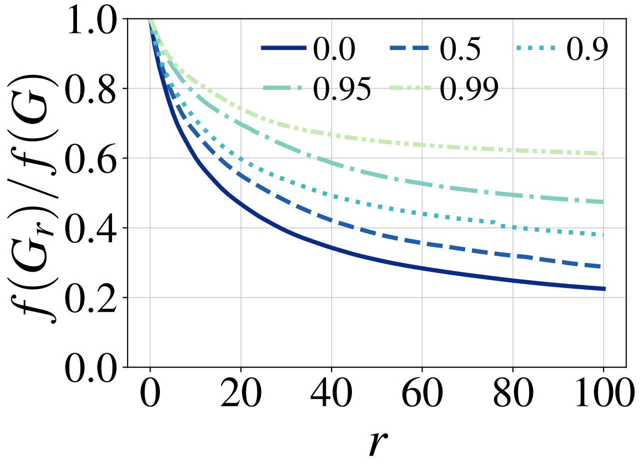


...outperforms all *baselines*

Gamine...

...is robust to changes in the *quality threshold*





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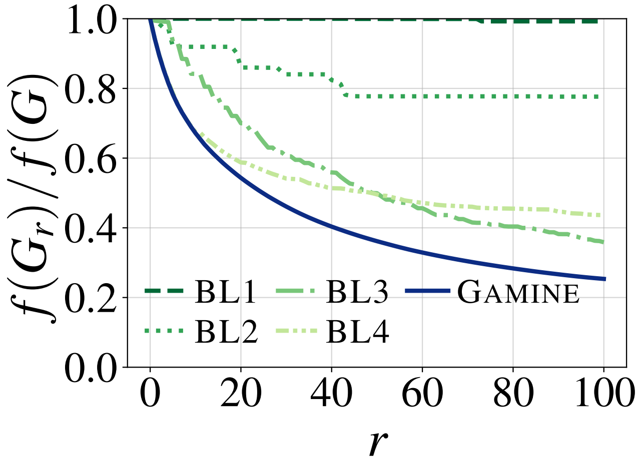

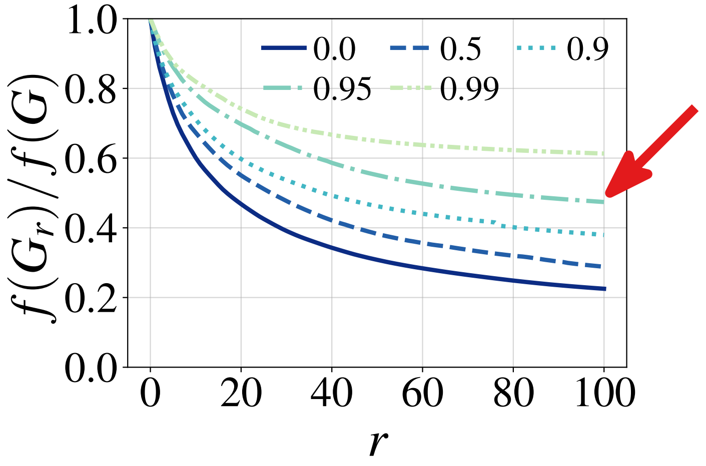

50% exposure reduction

5% drop in recommendation quality

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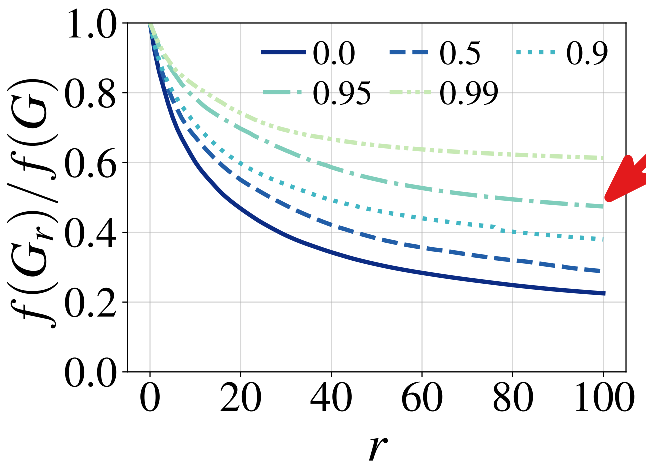

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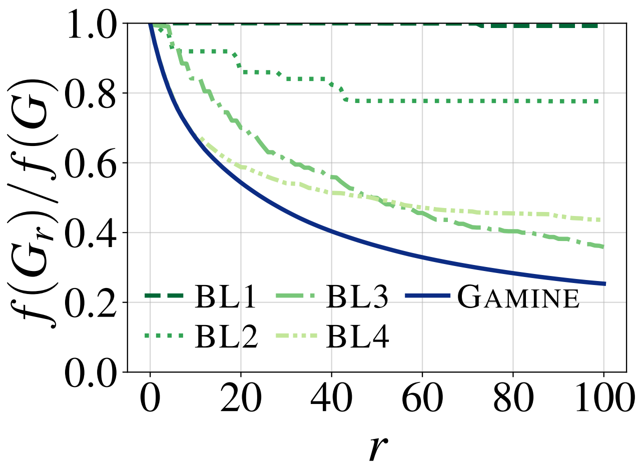

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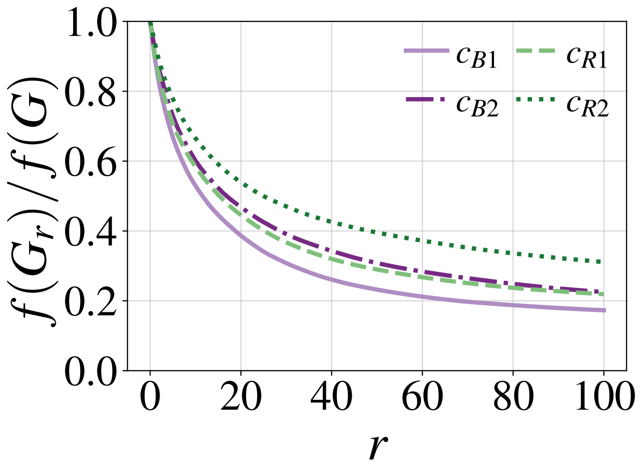
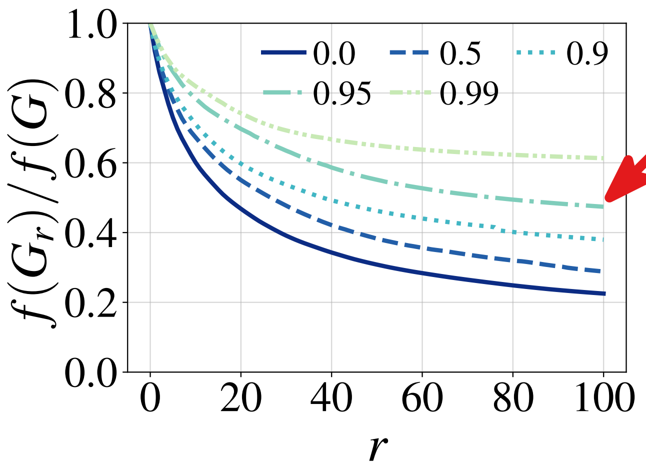


...is robust to changes in the *cost function*

Gamine…

…is robust to changes in the *quality threshold*



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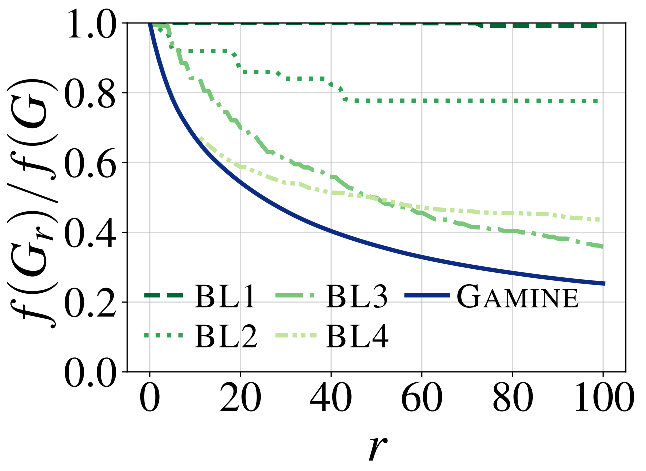
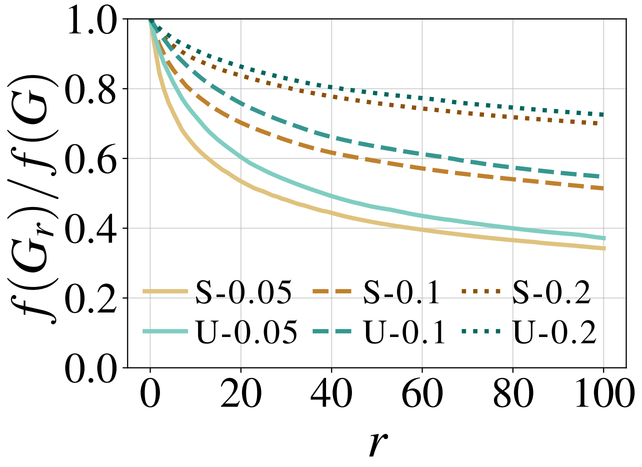
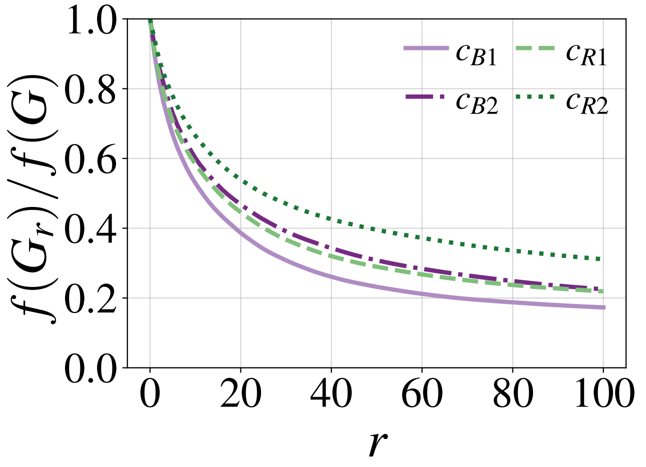


…outperforms all *baselines*

…is robust to changes in the *user model*





…is robust to changes in the *cost function*

GAMINE…

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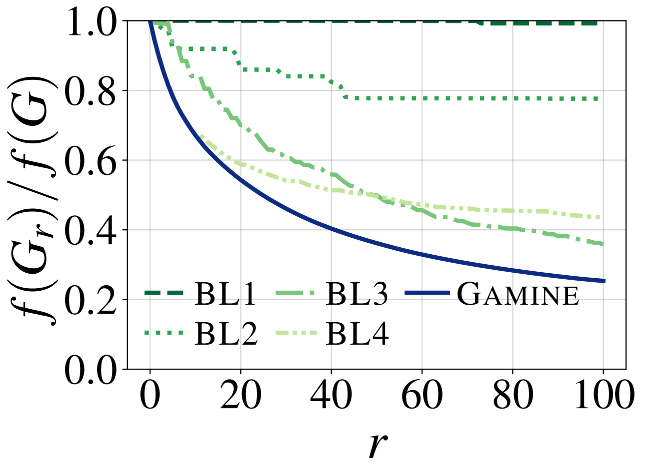

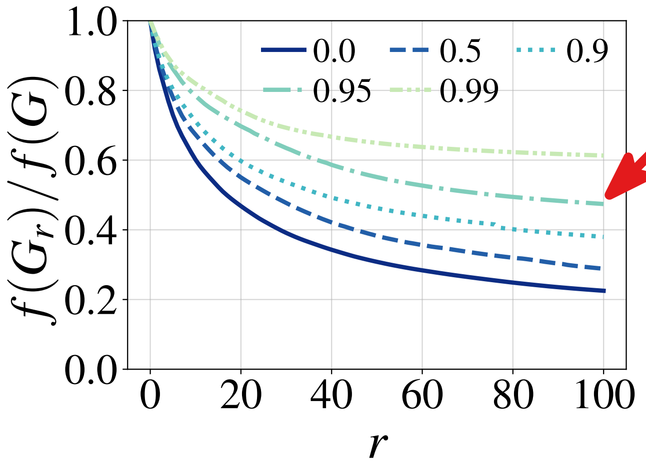

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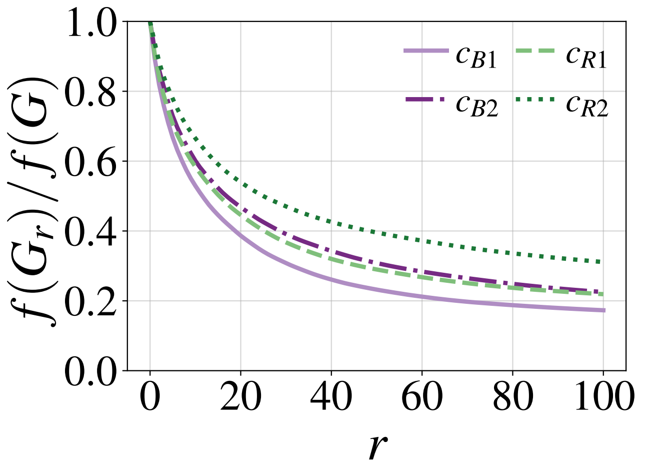
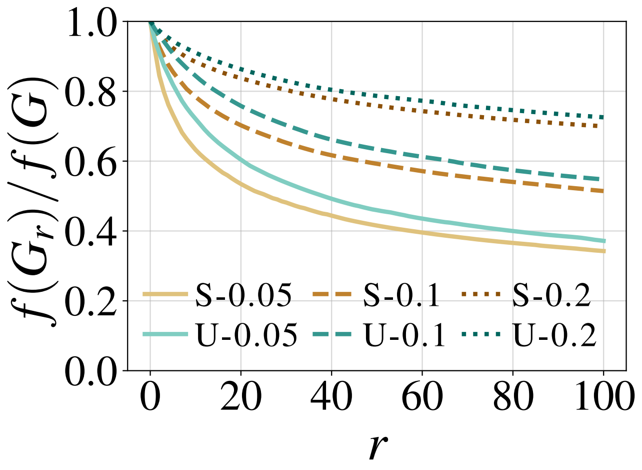
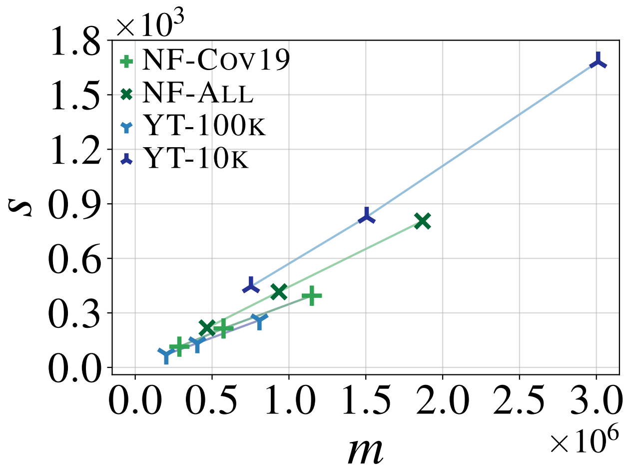


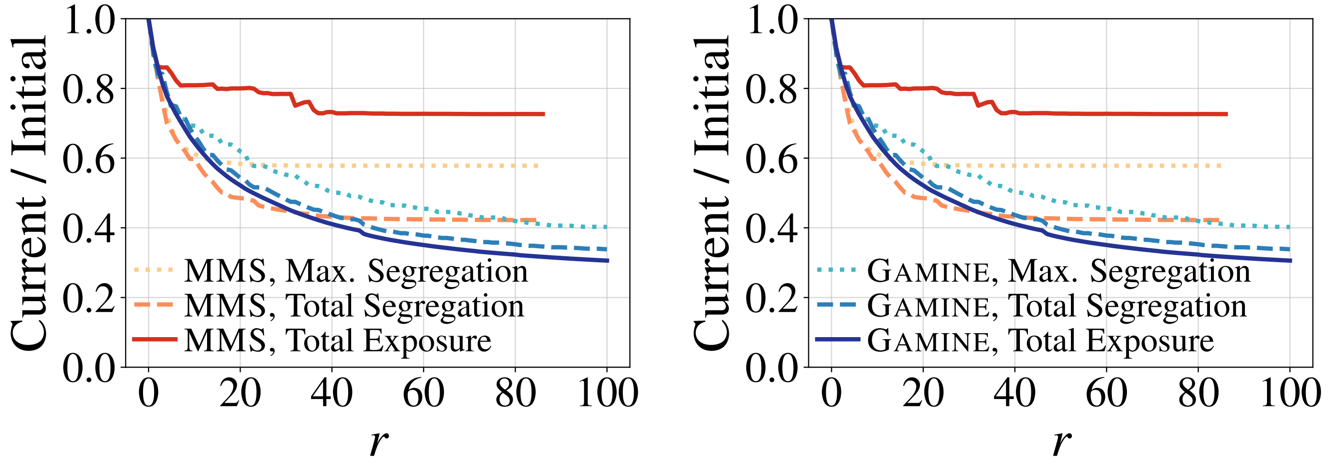




…is robust to changes in the *cost function*

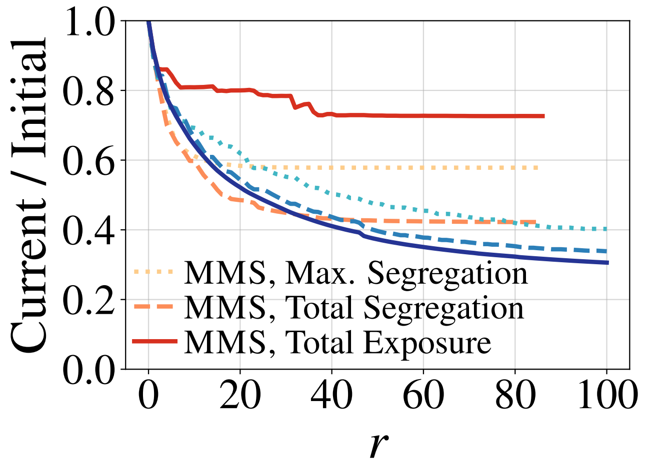
…and scales linearly

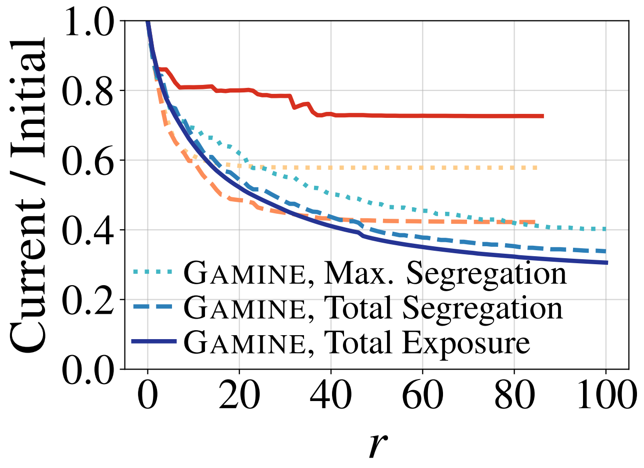
Gamine…



…outperforms MMS on the video data and…

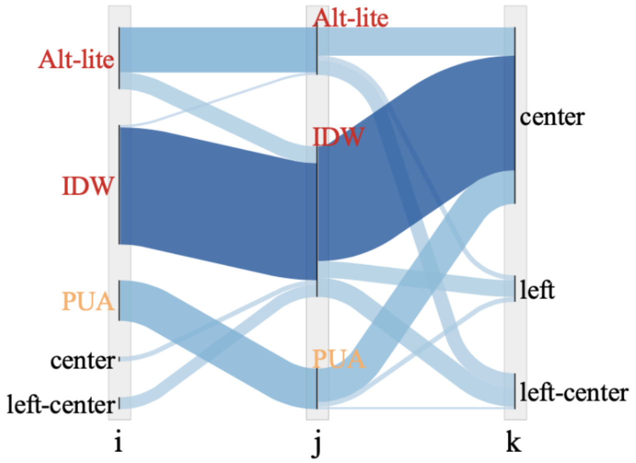
Gamine...

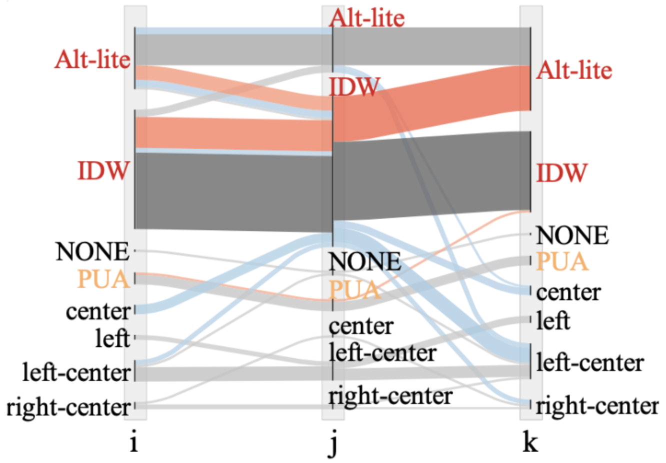




...outperforms MMS on the video data and...

...generates *interpretable* rewirings, but...





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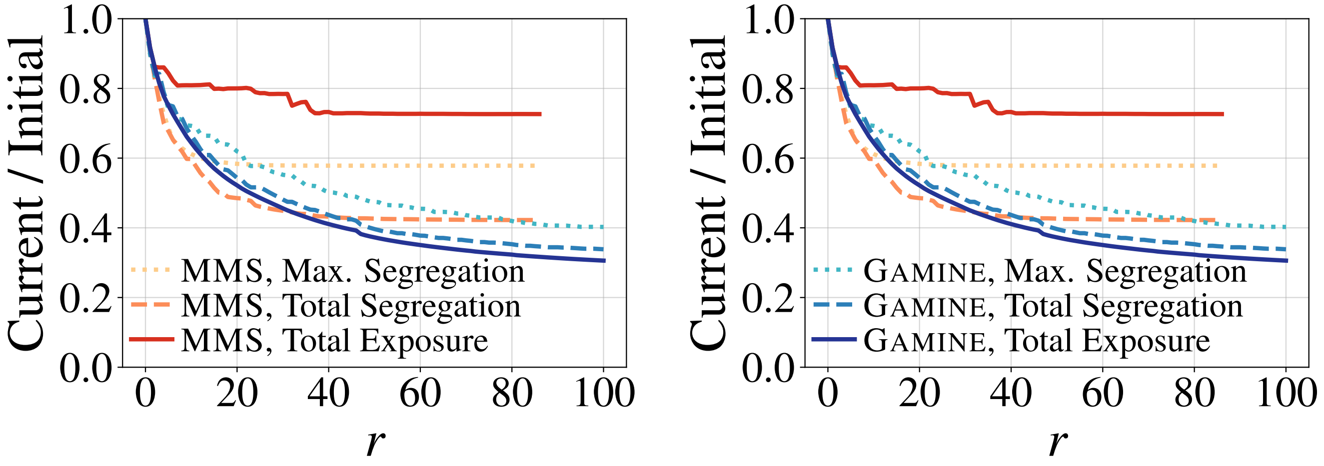

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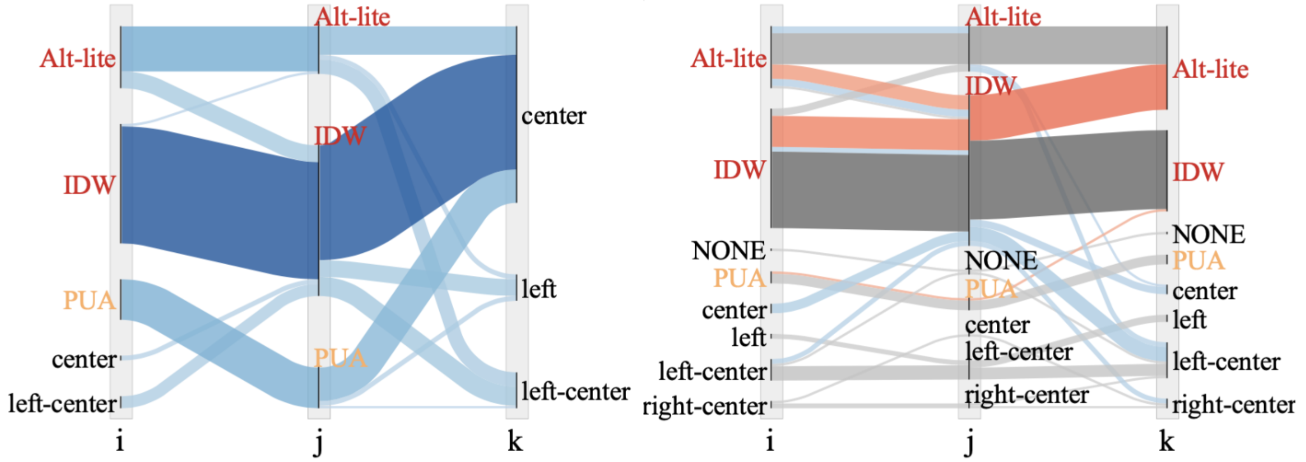

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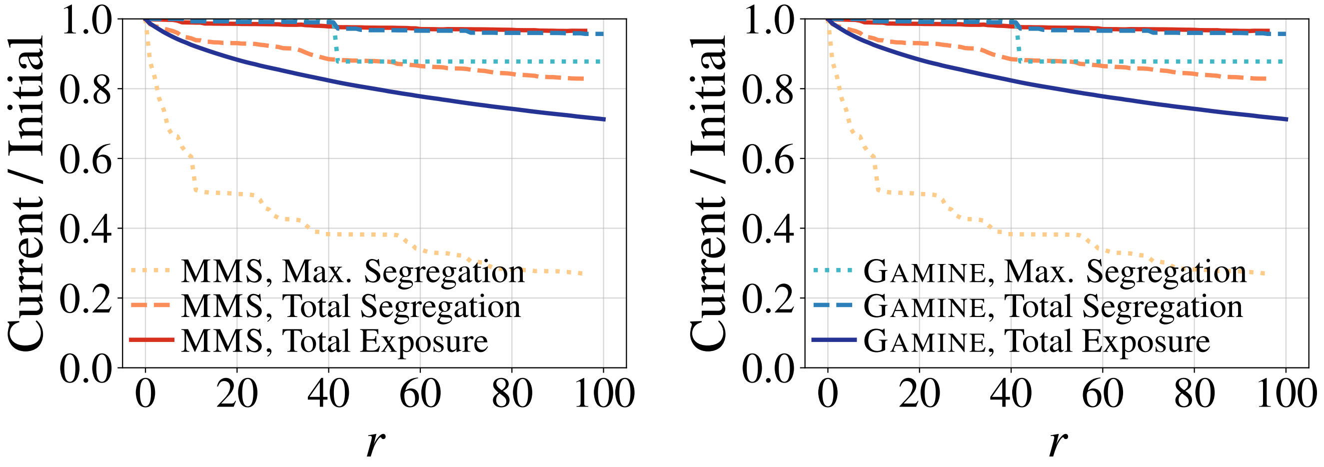

...it doesn't do nearly as well on the news data...

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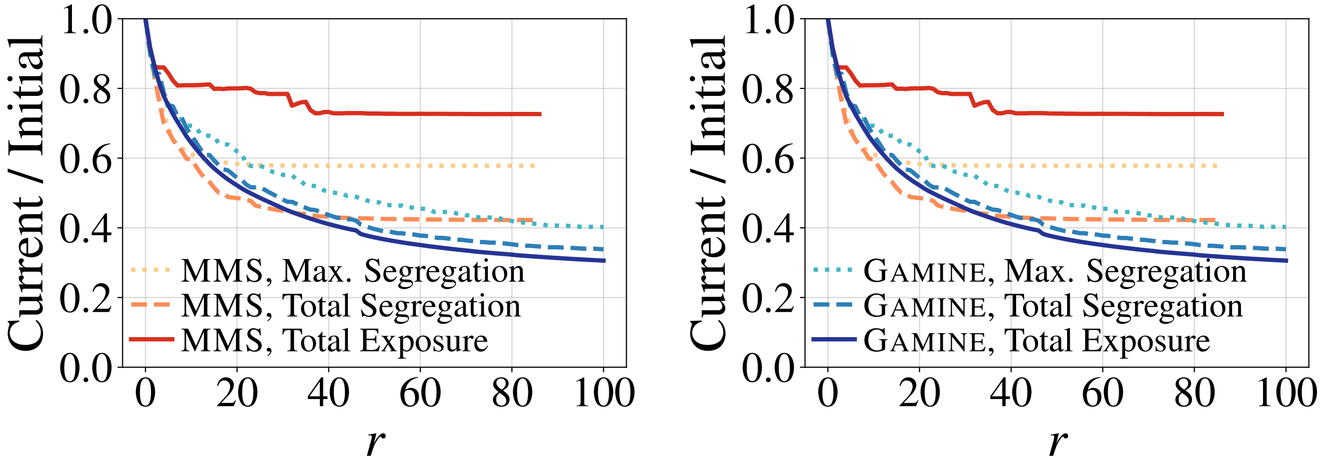

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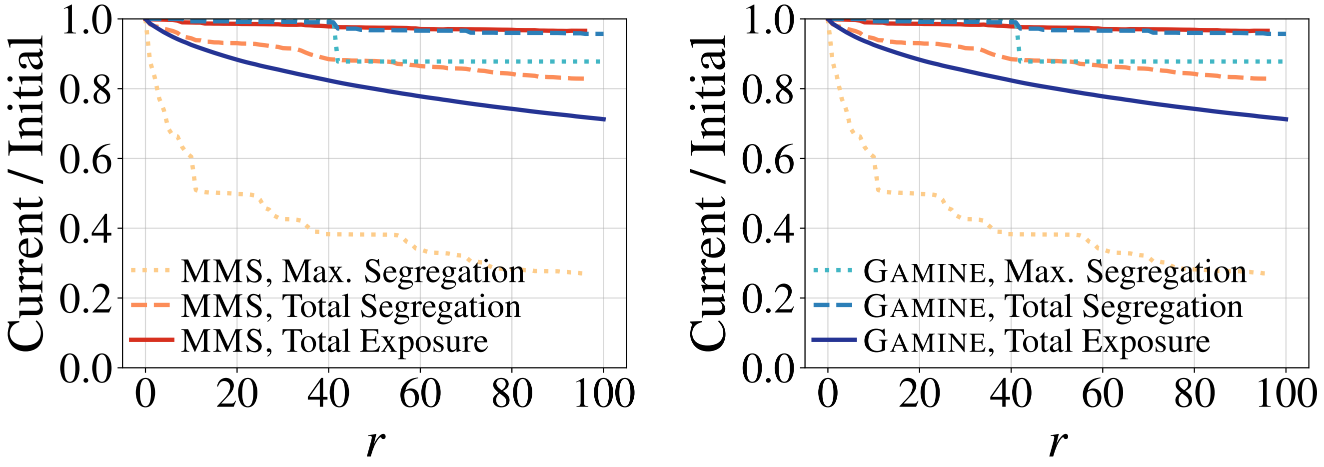

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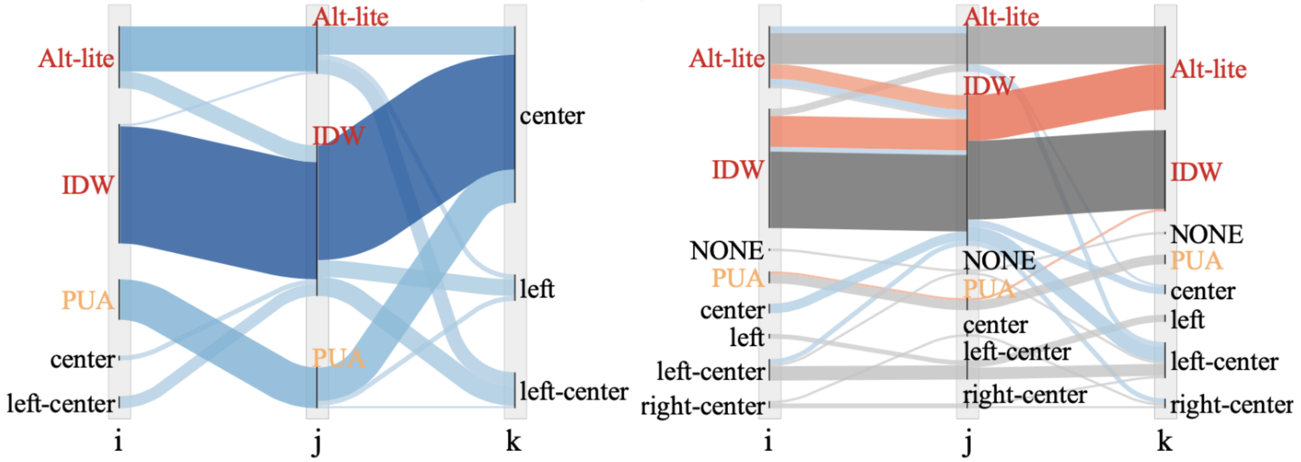
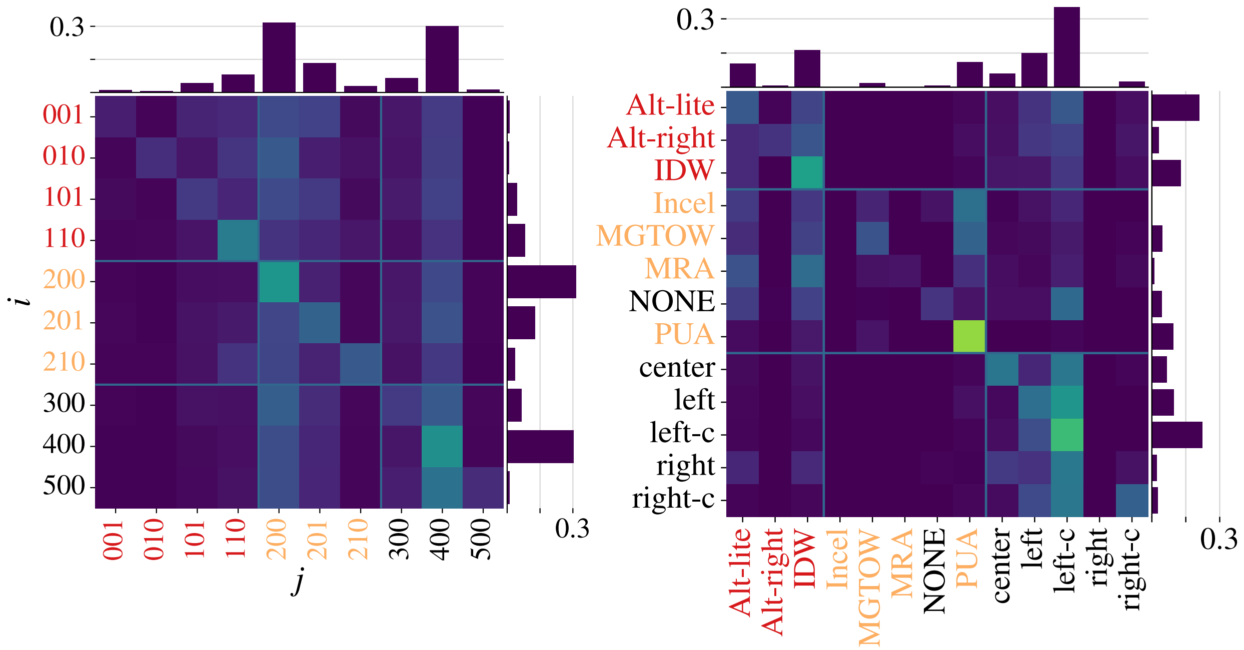


…because the news data is more *complex*!



Summary

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Real-world problem + modeling assumptions → r-rewiring exposure minimization (REM)

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= Optimizing random walks via graph editing under budget constraints

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Favorite future work: *Data complexity*

Thank you!

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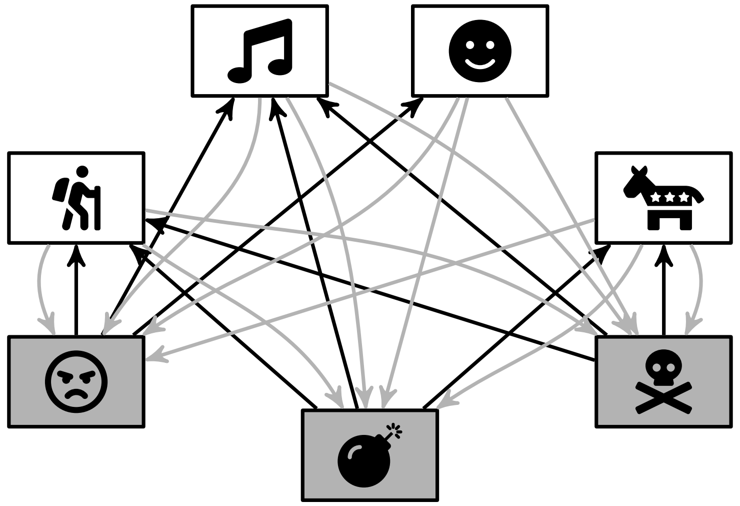

Conditional Submodularity

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Partition V into two classes

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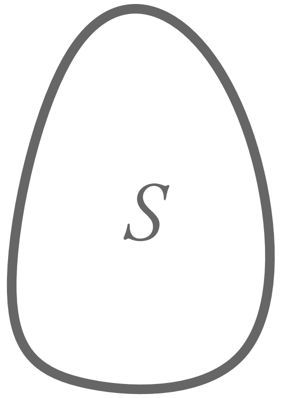
Safe nodes $S = \{i \in V \mid \mathbf{e}_i^T \mathbf{F}\mathbf{c} = 0\}$



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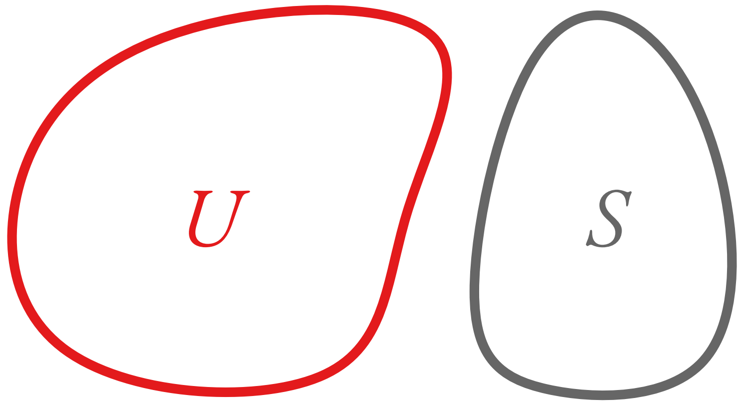

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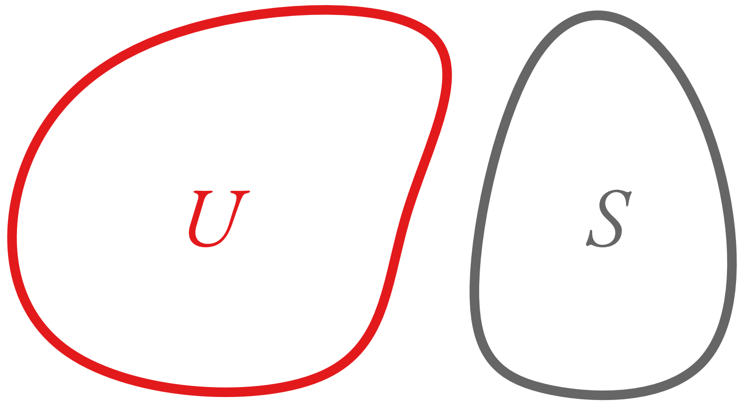
Partition E into four classes based on S and U



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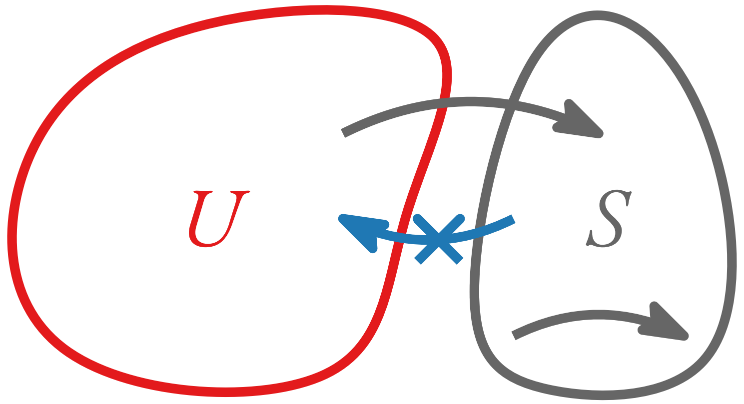
Partition E into four classes based on S and U

$E_{AB} = \{(i,j) \in E \mid i \in A \wedge j \in B\}$



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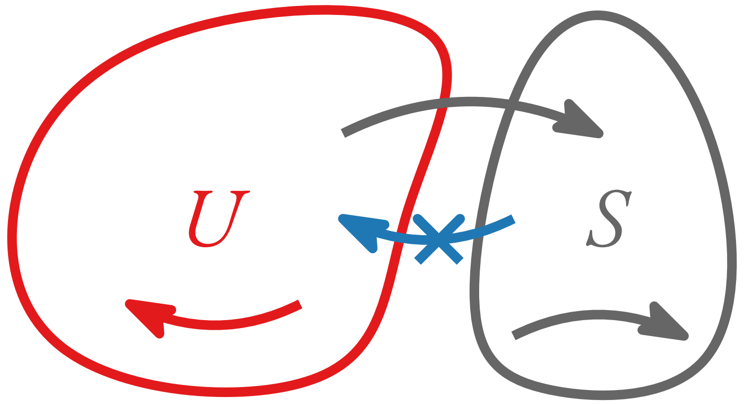

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$E_{AB} = \{(i, j) \in E \mid i \in A \wedge j \in B\}$

Only problematic class: $E_{UU} = \{(i, j) \mid i \in U \wedge j \in U\}$

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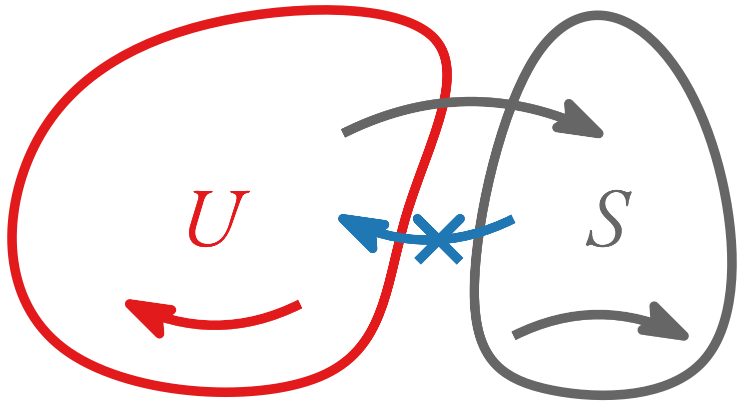

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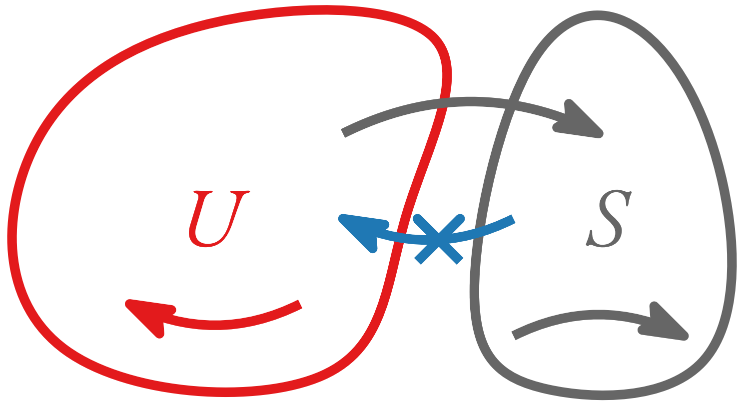
Define $\Lambda^+ = \max\{\delta^+(i) \mid i \in U\}$

Conditional Submodularity



Partition V into two classes

Safe nodes $S = \{i \in V \mid \mathbf{e}_i^T \mathbf{F}\mathbf{c} = 0\}$

Unsafe nodes $U = \{i \in V \mid \mathbf{e}_i^T \mathbf{F}\mathbf{c} > 0\}$

Partition E into four classes based on S and U

$E_{AB} = \{(i,j) \in E \mid i \in A \wedge j \in B\}$

Only problematic class: $E_{UU} = \{(i,j) \mid i \in U \wedge j \in U\}$

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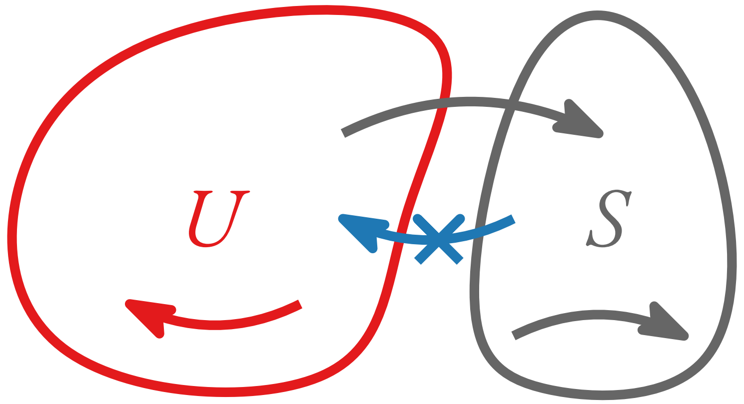

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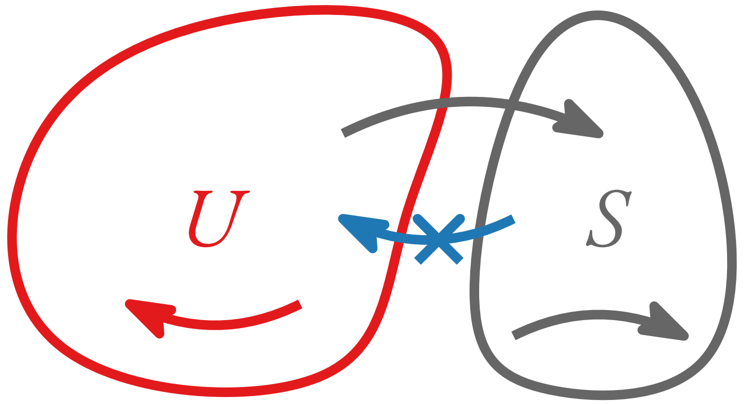

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Intuition: Pick edges from E_{UU} to rewire into S

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Greedy step $\leftrightarrow$ Rank-one update of transition matrix $\mathbf{P}$

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→ A rewiring $(i, j) \rightarrow (i, k)$ is *greedily permissible* iff $\tau > 0 \leftrightarrow j$ is more exposed than k

NP-Hardness: Setup

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Reduction from Minimum Vertex Cover

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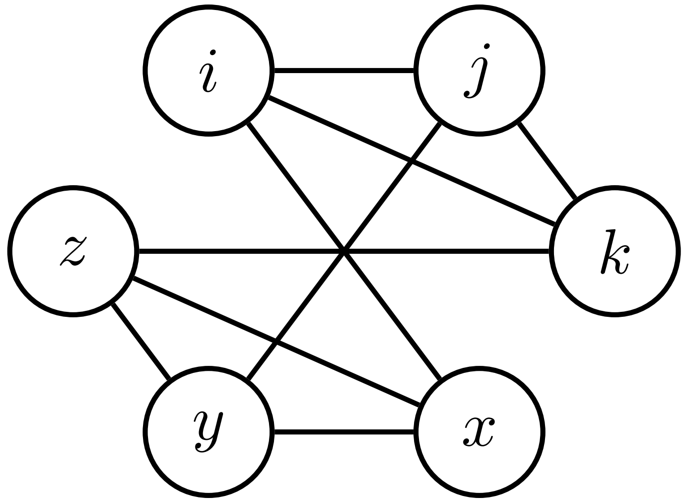
Reduction from Minimum Vertex Cover



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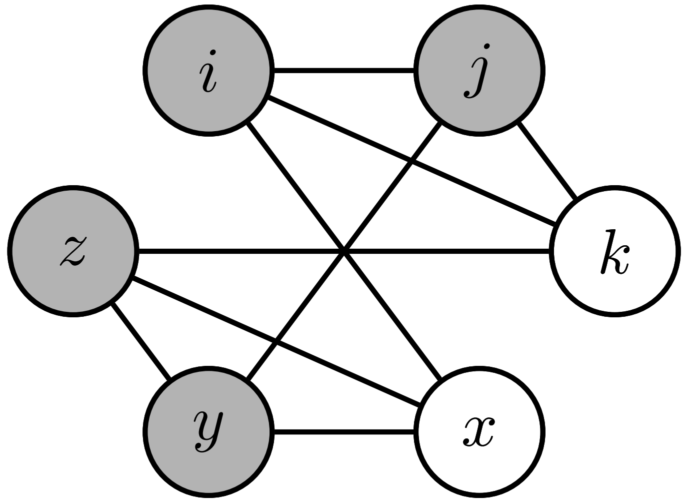

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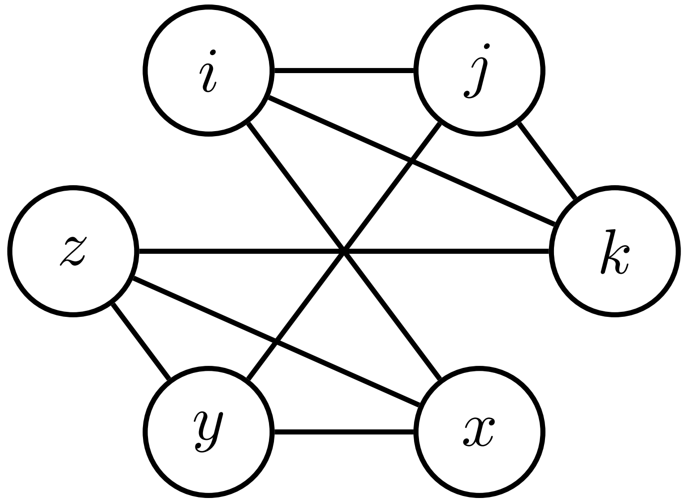

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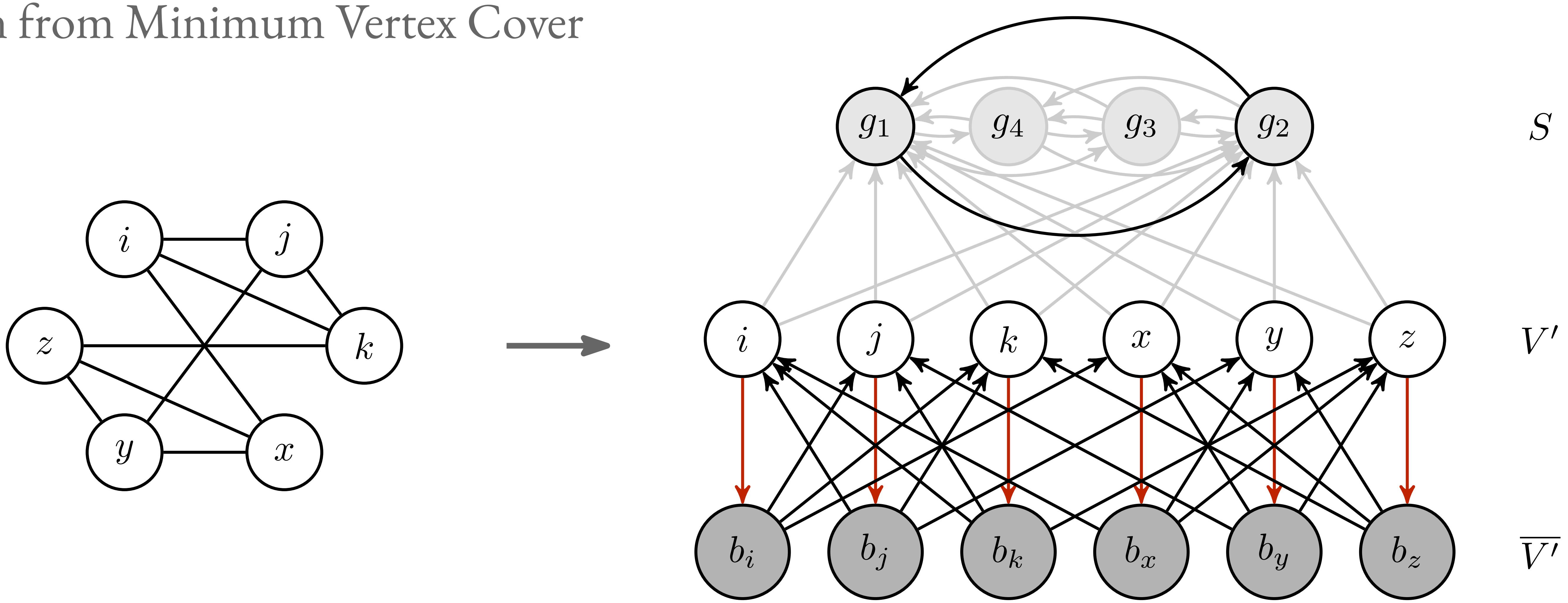

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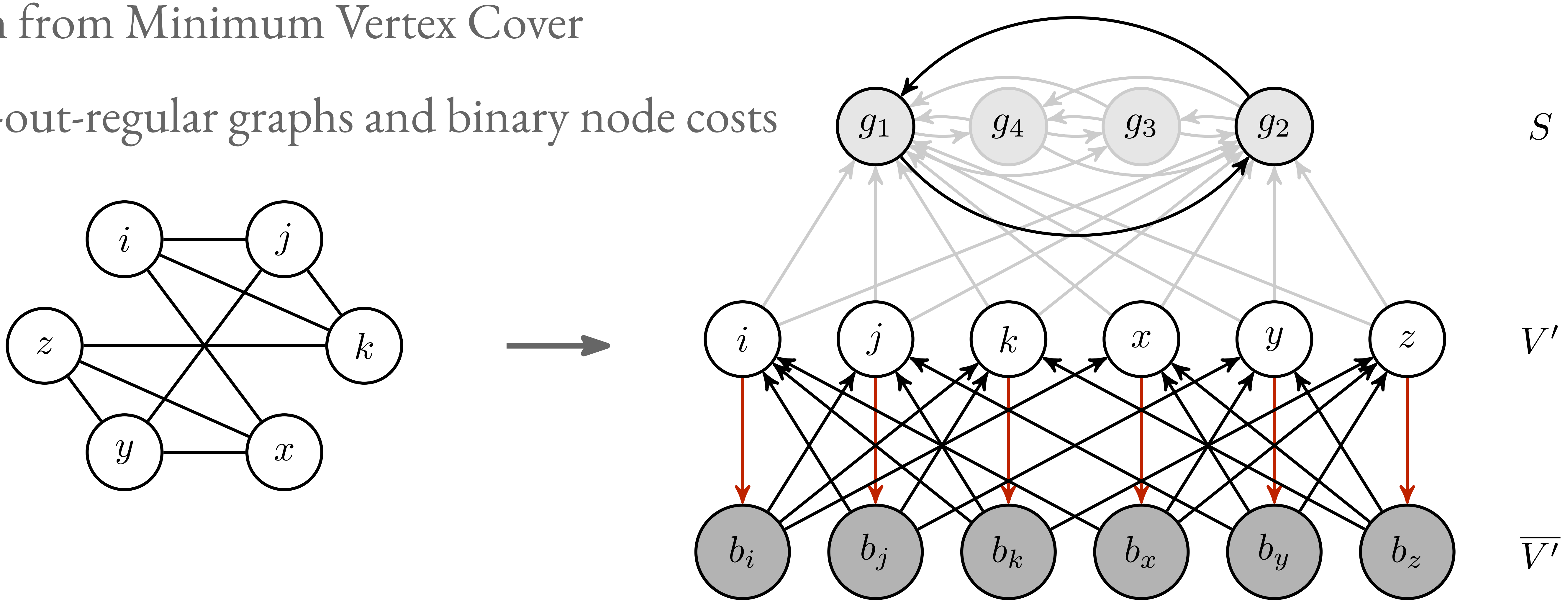
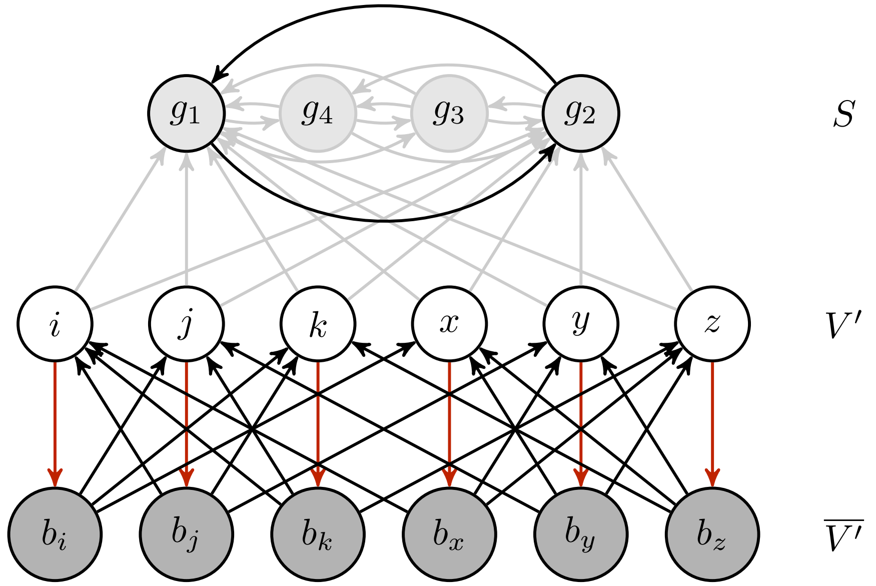
Reduction from Minimum Vertex Cover

Even for 3-out-regular graphs and binary node costs



NP-Hardness: Intuition



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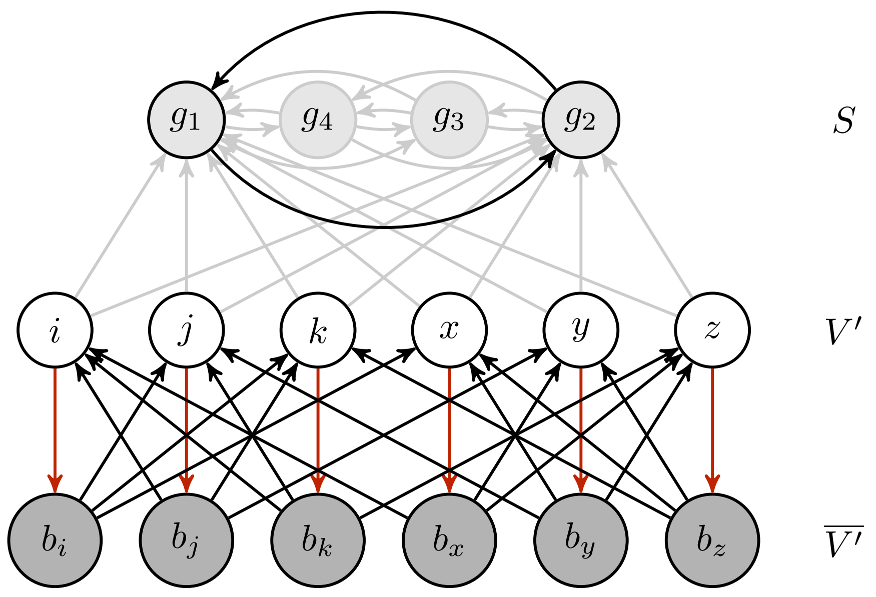
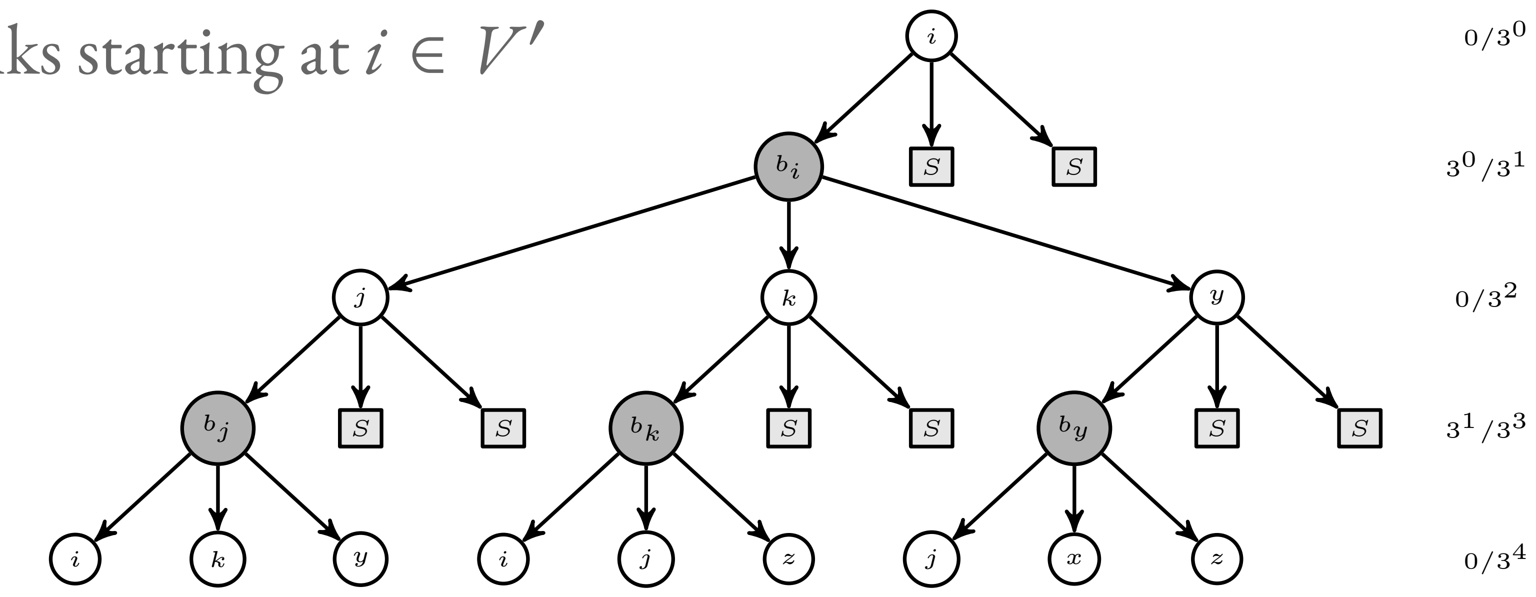


Walks starting at $i \in V'$



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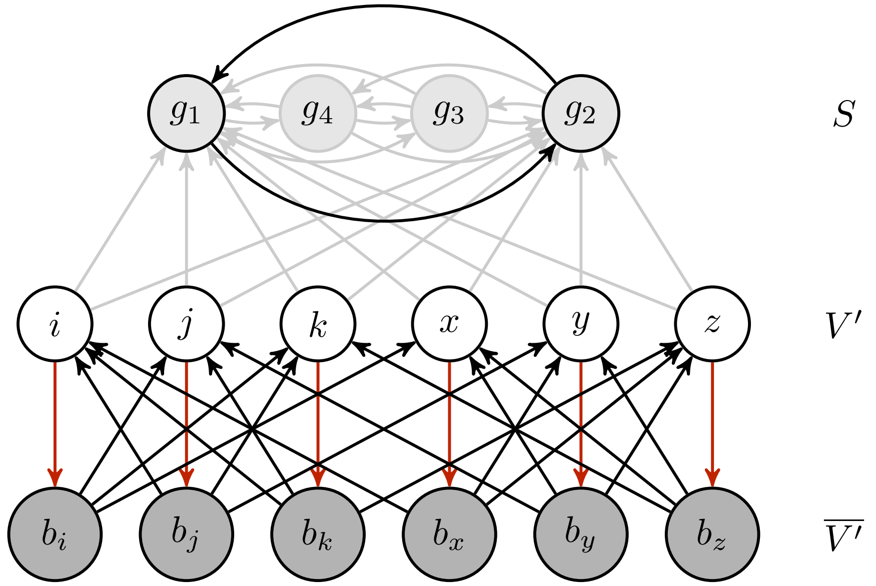

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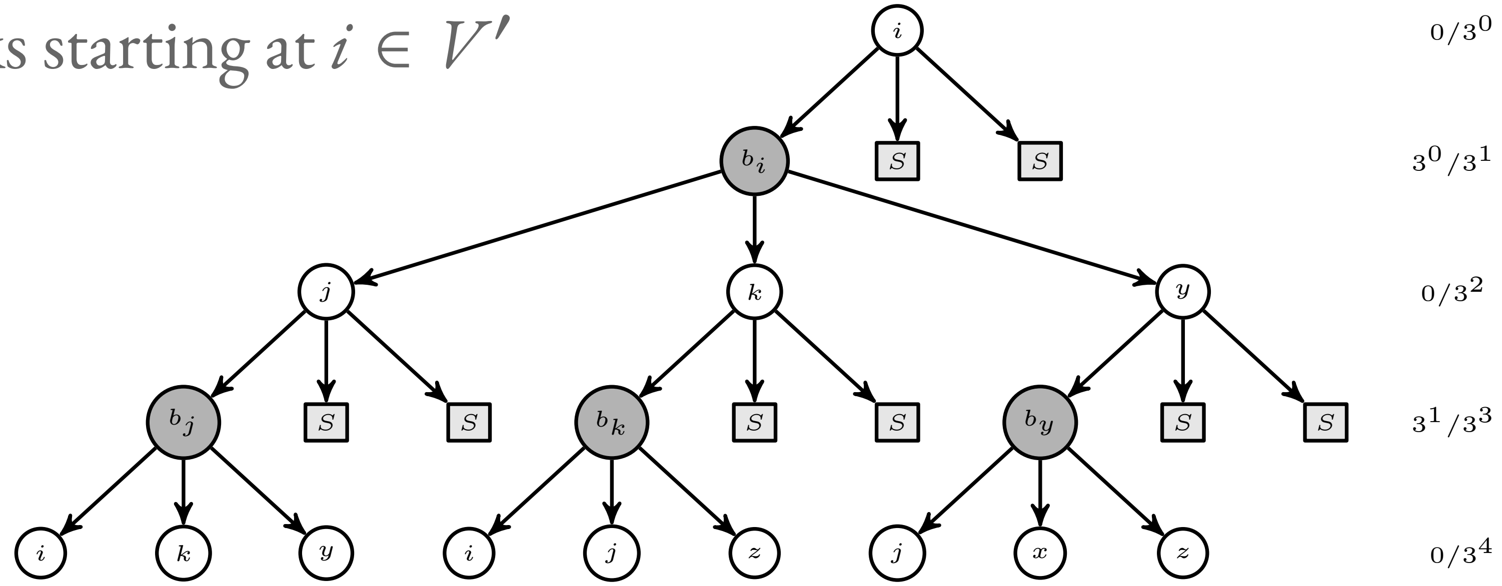

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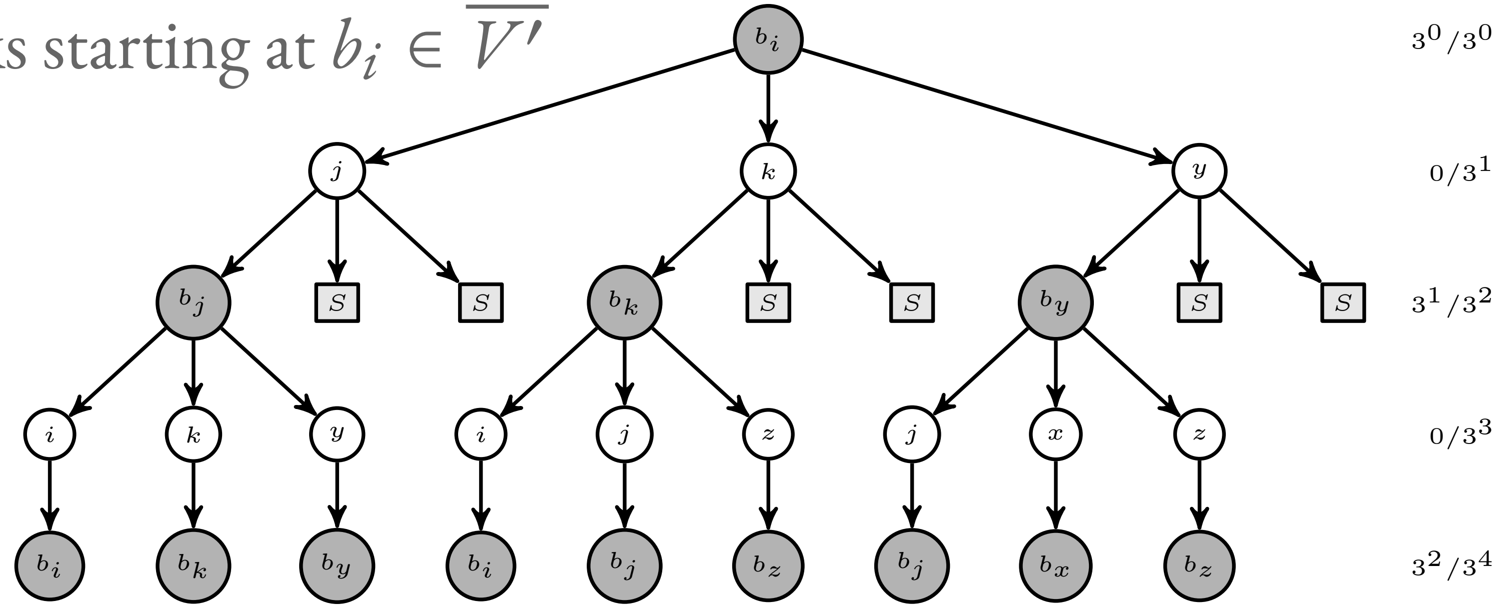

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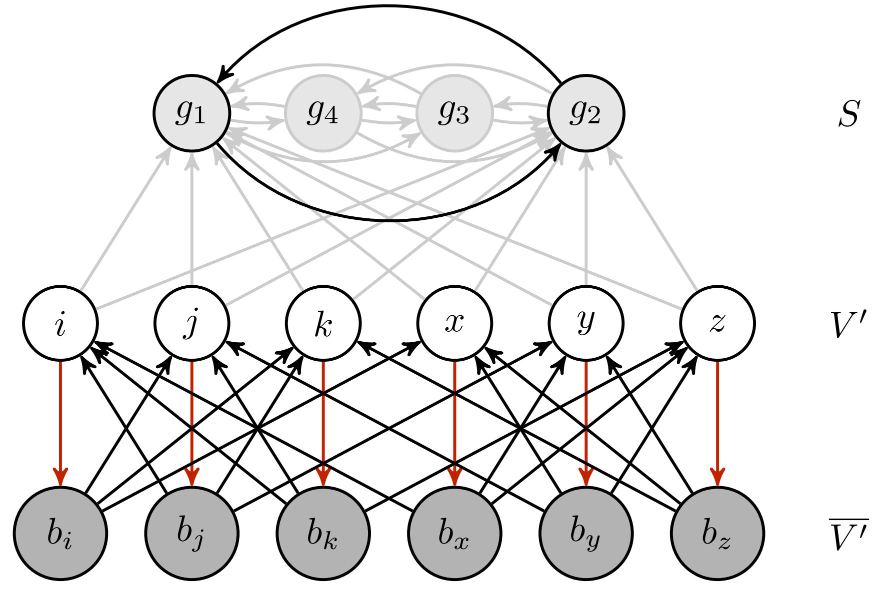

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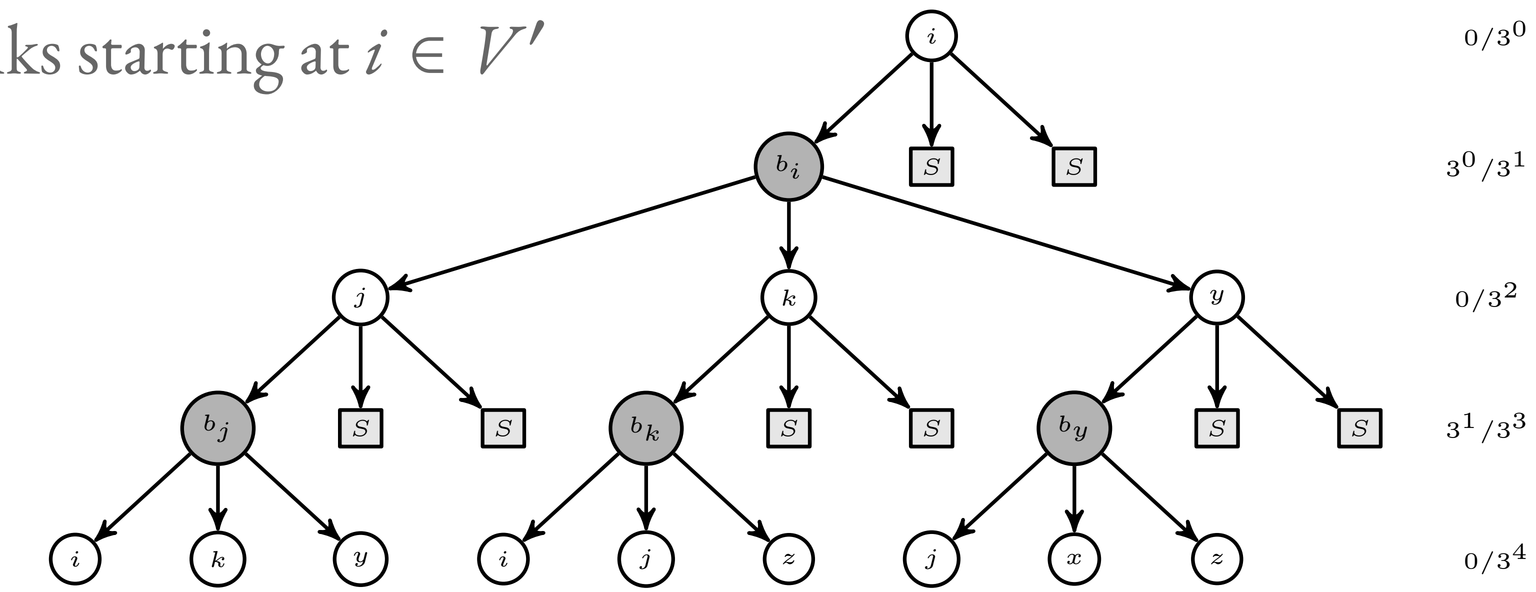

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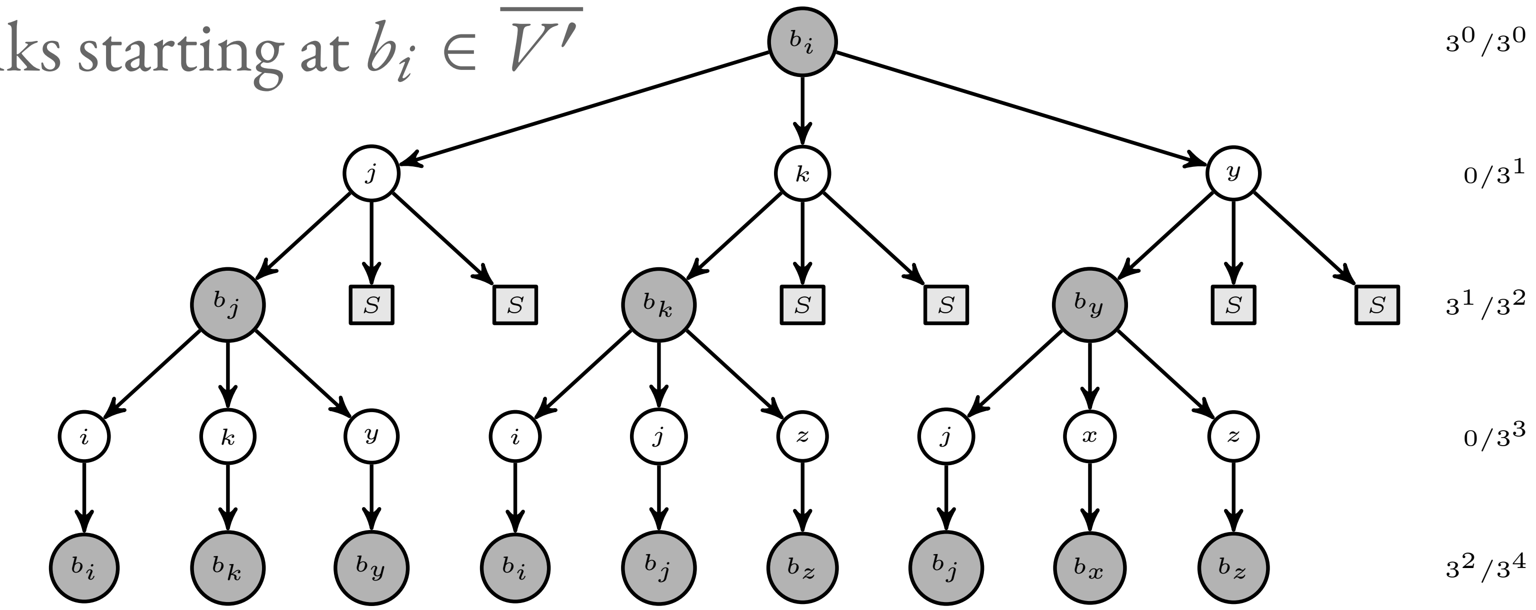

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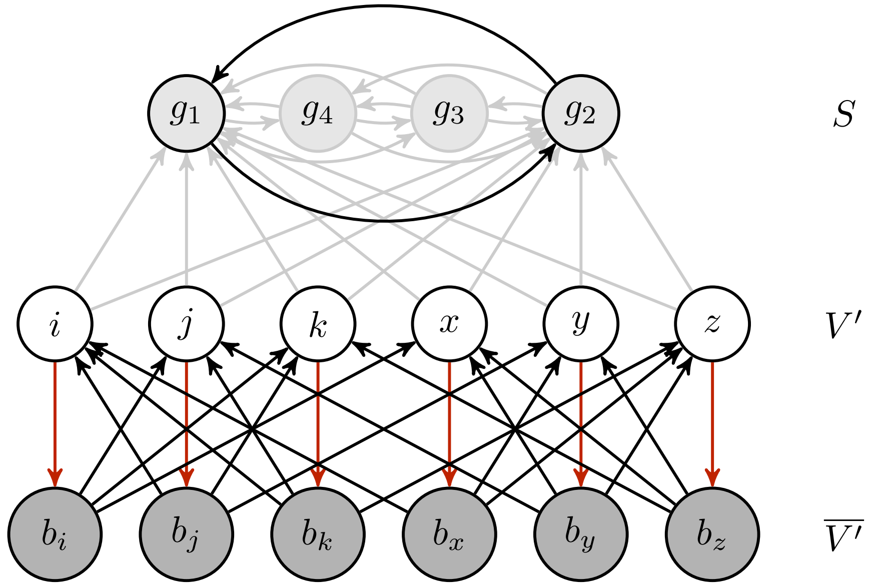

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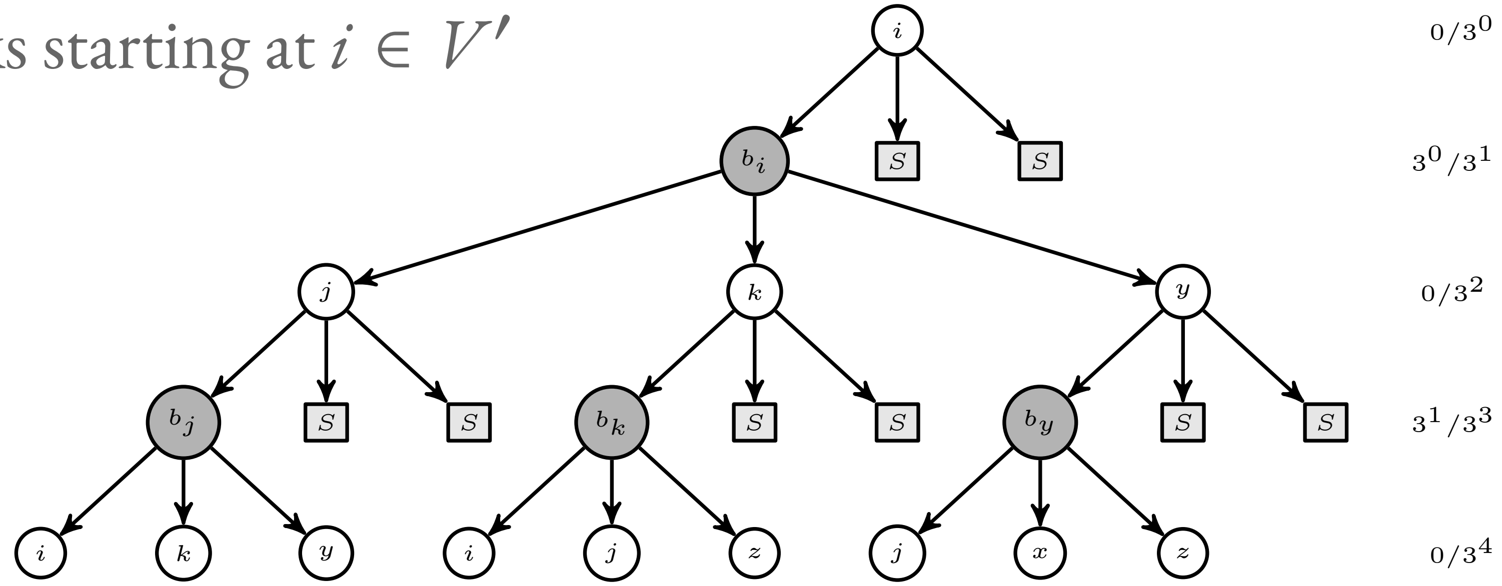

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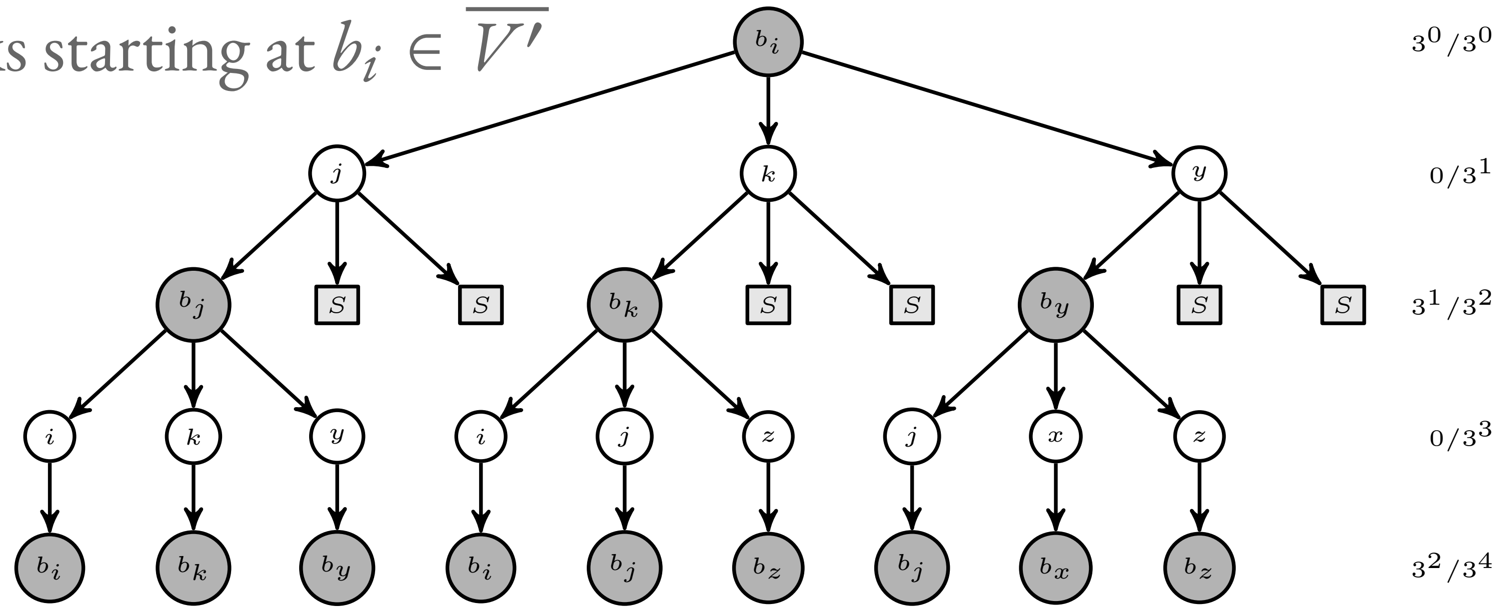

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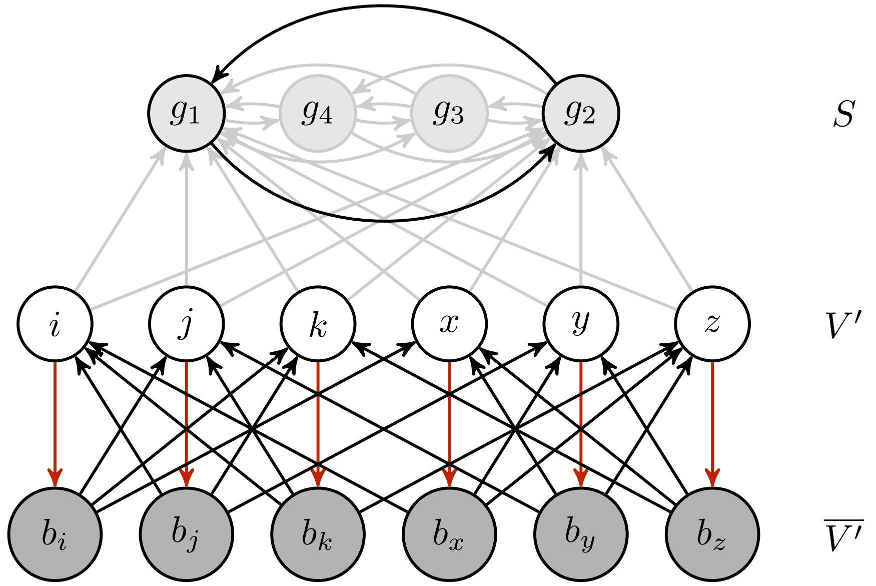
NP-Hardness: Sketch



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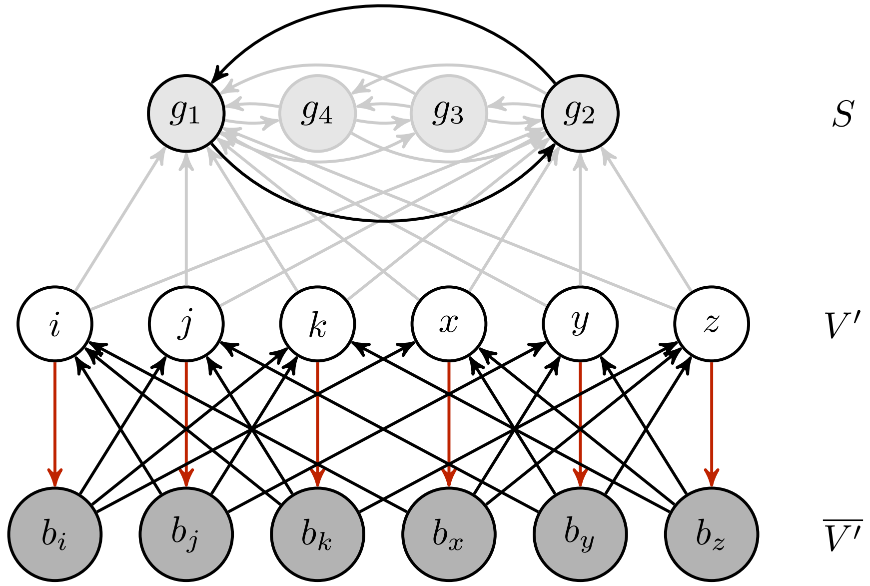

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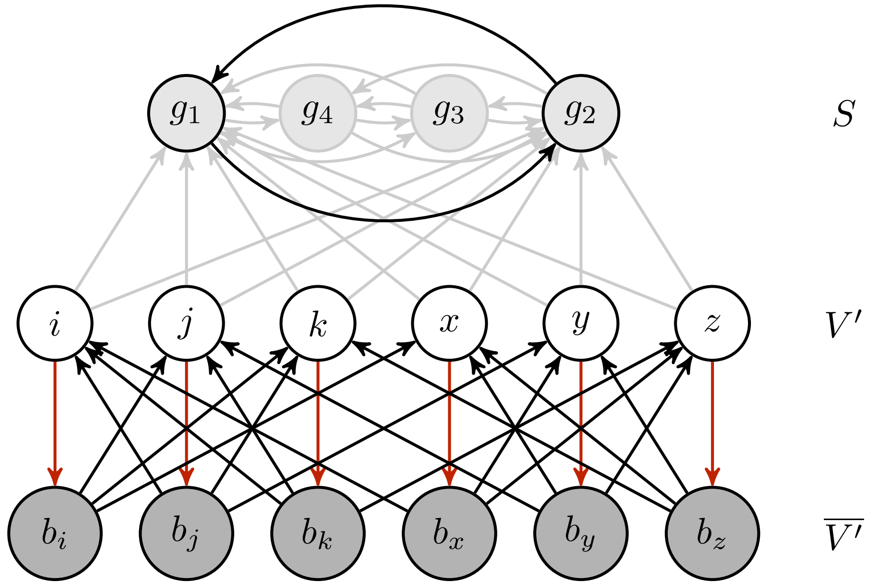

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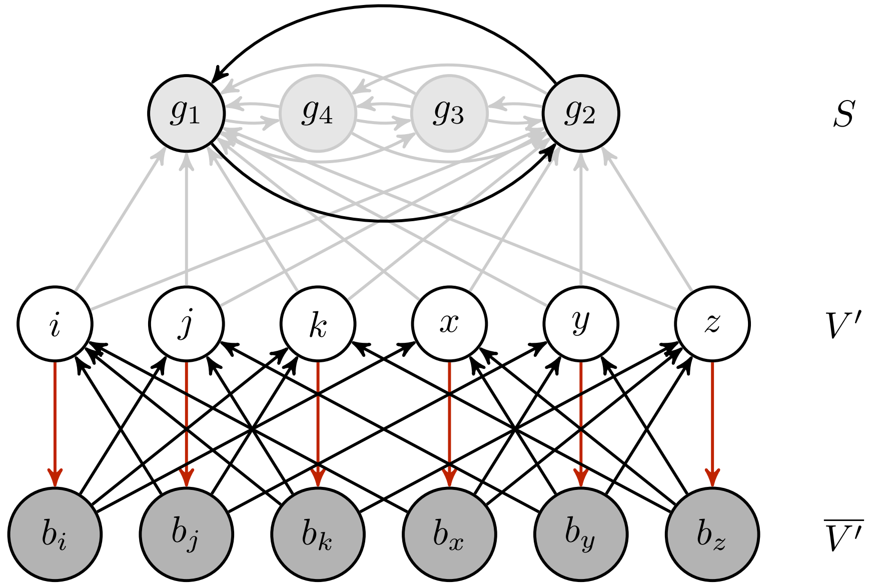

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Rewiring $(i, b_i) \to (i, g_n)$ in $G \leftrightarrow i \in \mathrm{VC}(G)$

Reduction of $f(G)$ by individual rewiring

$$\frac{1}{3}(1-\alpha) + \frac{1}{9}(1-\alpha)^2 + \frac{2\gamma'}{27}(1-\alpha)^3 + \varepsilon'$$

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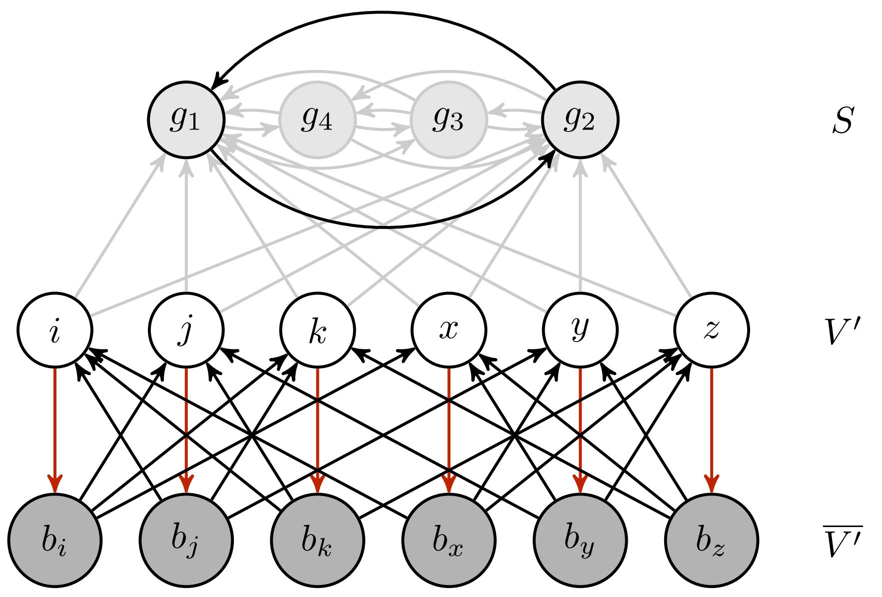

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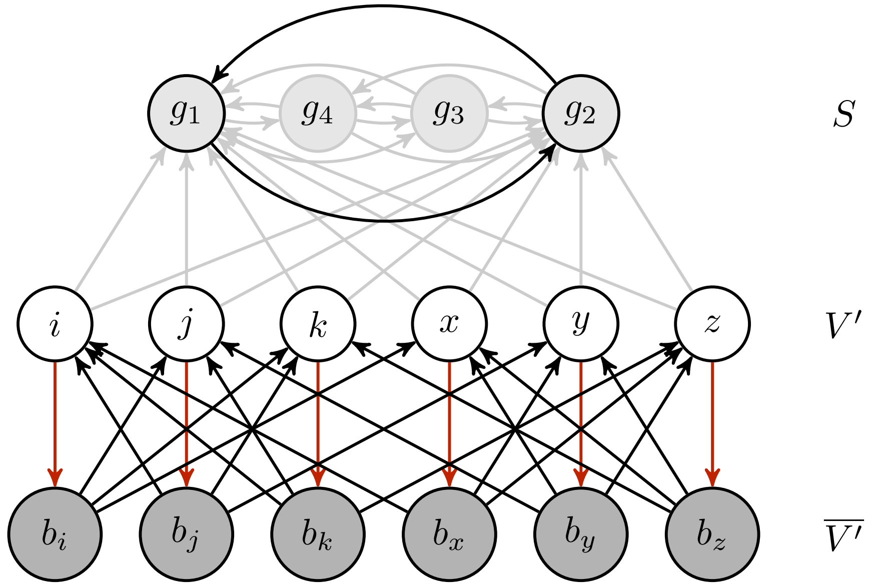

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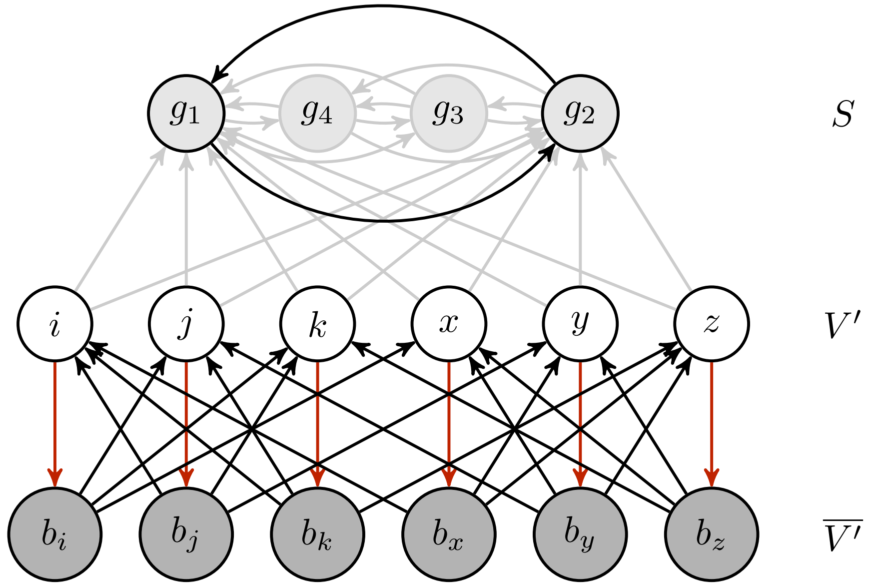

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MVC of size at most r

$$\leftrightarrow \; f_\Delta(G, G_r) = \tfrac{r}{3}(1-\alpha) + \tfrac{3r}{9}(1-\alpha)^2 + \tfrac{2m'}{27}(1-\alpha)^3 + \varepsilon$$

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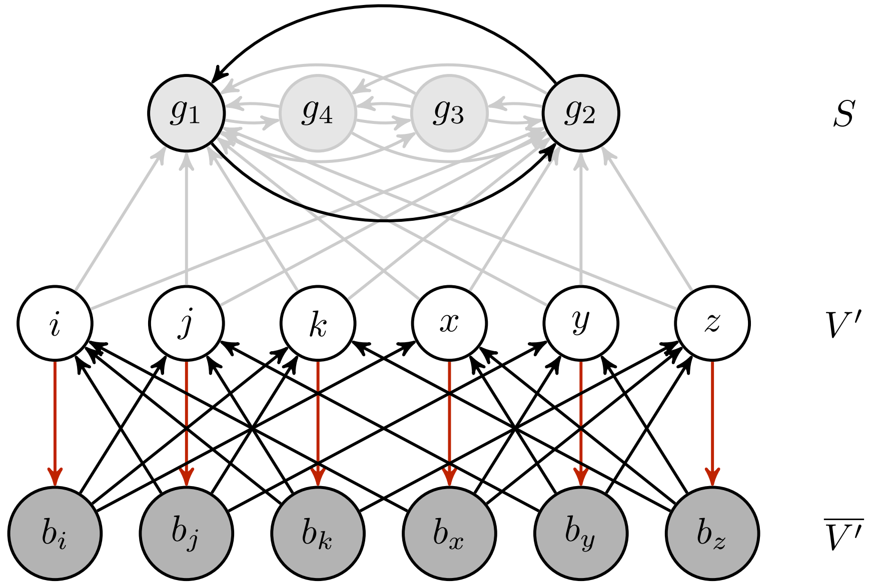

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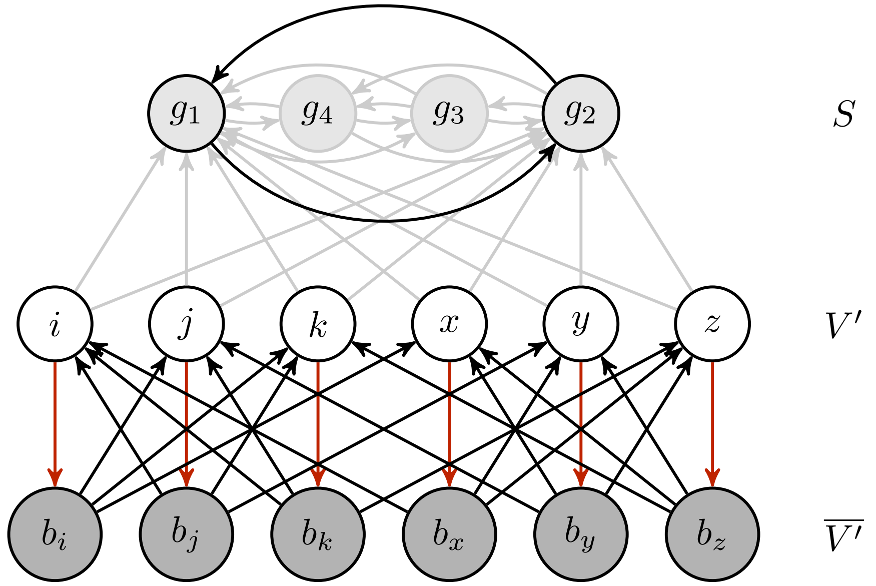
For $0.5 \leq \alpha \leq 1$, we have $\xi < \frac{2}{27}(1-\alpha)^3$

NP-Hardness: Sketch



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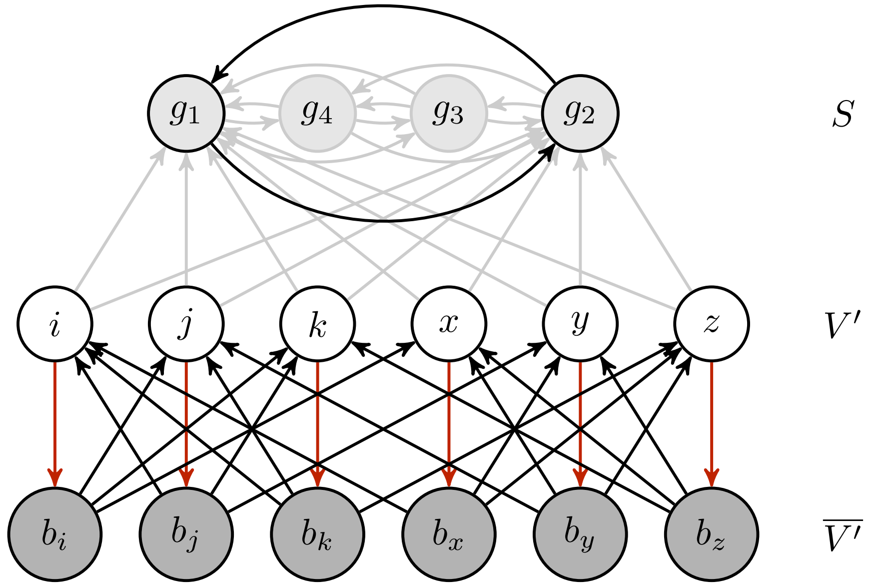
MVC of size at most r

$$\leftrightarrow \; f_\Delta(G, G_r) = \frac{r}{3}(1-\alpha) + \frac{3r}{9}(1-\alpha)^2 + \frac{2m'}{27}(1-\alpha)^3 + \varepsilon$$

For $0.5 \leq \alpha \leq 1$, we have $\xi < \frac{2}{27}(1-\alpha)^3$

ξ: *All* harm beyond step 3

NP-Hardness: Sketch



$$f(G) = \underbrace{n' \sum_{t=0}^{\infty} 3^{-t}(1-\alpha)^{2t}}_{\text{Contributions from } \overline{V'}} + \underbrace{n' \sum_{t=0}^{\infty} 3^{-t-1}(1-\alpha)^{2t+1}}_{\text{Contributions from } V'}$$

Candidate edges to rewire are $\{(a, b_a) \mid a \in V'\}$

Rewiring $(i, b_i) \to (i, g_n)$ in $G \leftrightarrow i \in \mathrm{VC}(G)$

Reduction of $f(G)$ by individual rewiring

$$\tfrac{1}{3}(1-\alpha) + \tfrac{1}{9}(1-\alpha)^2 + \tfrac{2\gamma'}{27}(1-\alpha)^3 + \varepsilon'$$

γ': Number of newly covered edges in G'

ε': Contributions of walks beyond step 3

MVC of size at most r

$$\leftrightarrow \quad f_\Delta(G, G_r) = \tfrac{r}{3}(1-\alpha) + \tfrac{3r}{9}(1-\alpha)^2 + \tfrac{2m'}{27}(1-\alpha)^3 + \varepsilon$$

For $0.5 \leq \alpha \leq 1$, we have $\xi < \frac{2}{27}(1-\alpha)^3$

ξ: *All* harm beyond step 3

→ Can check "truncated" $f(G_r)$ to decide MVC